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Mathematical Modeling and Implementation of the Airship Navigation

F. Jelenčiak*

** FernUniversität in Hagen, MI / PRT*
(Tel: +49 2331 / 987 1119.; e-mail: Frantisek.Jelenciak@fernuni-hagen.de)

Abstract: The main source of errors for airship navigation is that the airship body is not solid. For this reason a standard fixed calibration for a navigation system is not the best solution. This article provides an overview of the proposed navigation system for airships with compensation of errors due to resilience.

1. INTRODUCTION

Navigation systems and flight control are basic equipments of unmanned aerial systems (UAS). For modeling of the navigation system (see references) it is necessary to select appropriate coordinate systems and to select description methods for body orientation. The next step is to apply transformations between coordinate frames, and to relate them to kinematics and dynamics theory. Stability and reliability of the navigation system can be achieved by combining the on-board IMU (Inertial Measurement Unit) with external observation units such as GPS (Global Positioning System), barometric and magneto-compass units.

The proposed model of airship navigation uses coordinate systems like ECEF (Earth-Centered-Earth-Fixed) and NED (North-East-Down). For the description of the airship's orientation Euler angles as well as roll, pitch, and yaw are used. All measured data are joined together by a special type of direct Kalman filter. This filter and the strapdown modification are designed in such a way that together with other measurement units it is possible to compensate the problem, which is caused by the non-solid body of airships. Navigation systems implemented on airships show characteristic errors due to this fact. The following facts can be considered as sources of these errors: an airship body changes its shape depending on several parameters, e.g. the helium pressure in the hull; the ambient temperature; relative position of inner and outer hull; aerodynamic forces. The shape changes are in general nonlinear. For these reasons a fixed calibration for navigation system is not the optimal solution. This article provides an overview of the proposed navigation system for the airship which compensates the aforementioned errors.

The development of the airship navigation algorithm was carried out in the programming environment Matlab-Simulink. The toolboxes used are "Target Support Package" and "Code Composer Studio" was designed Simulink-scheme, which is algorithm interpretation of the airship model navigation. This Simulink-scheme can be transferred as program code into the microprocessor TMS320F28335, which together with sensors of the IMU (Inertial Measurement Unit), barometric unit, GPS, and magneto-compass are core of the navigation system (http://prt.fernuni-hagen.de/ARCHIV/2010/fernsehen_2010.html, http://www.derwesten.de/staedte/hagen/FernUni-Luftschiff-auf-Minensuche-id2359263.html).

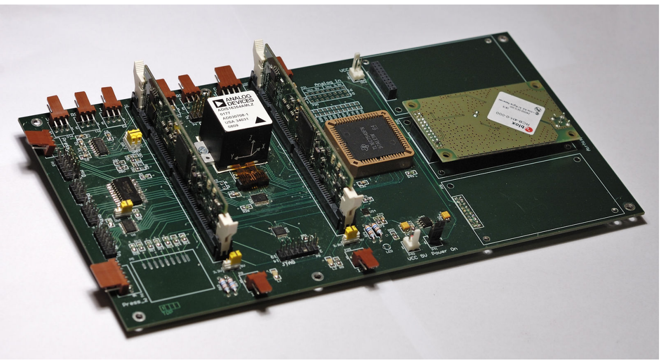

Fig. 1. Electronic board of the navigation system (design by Dr.-Ing. Ivan Masár*).

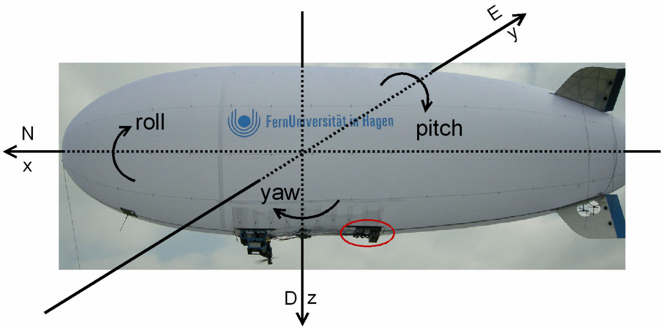

Fig. 2. Airship photo with selected area for storing of the navigation system (two hulls, length 9m, diameter 2,5m, volume $27m^3$).

The article is structured as follows: In chapter 2, the basic scheme of the navigation algorithm is shown. In chapters 3-7, the essential parts of the navigation algorithm are described in detail. In the final chapter 8, conclusions and results are given.

2. BASIC SCHEME

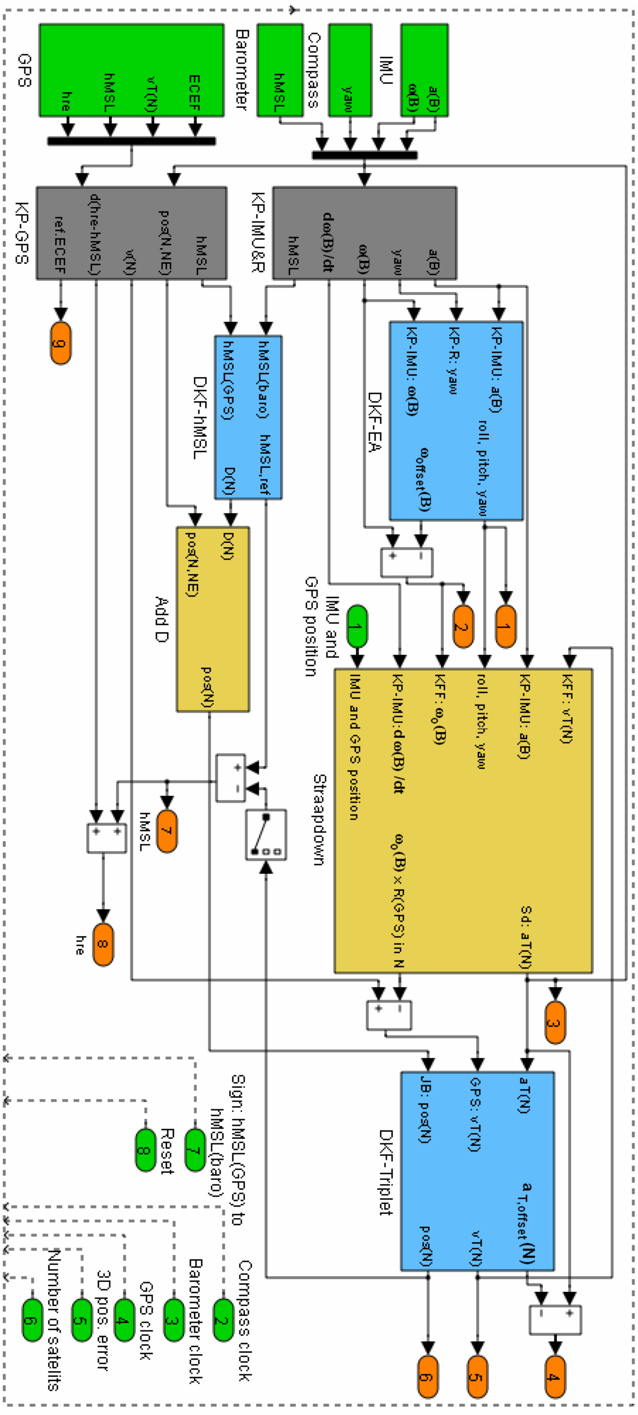

Fig. 3. Basic scheme of the navigation algorithm.

At the beginning it is necessary to divide the input navigation data into two groups. The first group represents data from the IMU. The second group represents data from other sensors.

From the IMU (strapdown platform), it is possible to load data such as acceleration and angular velocity of rotation. The advantage of these data is that they may be available at high sampling rates with relatively high sensitivity and accuracy. The disadvantage of these data is, that velocity, position or orientation determined by direct time integration are loaded with error, which in time always increases.

A second group is represented by data from GPS, barometer, and magneto-compass. The advantage of these data is, that the error with which they are loaded is not dependent on time. The disadvantage of these data is that they often show less accuracy, sensitivity, and their availability with a smaller sampling rate. Advantages of both groups can be obtained by some form of Kalman filter.

3. SIGNALS AND BLOCKS

In Fig. 3, all inputs into the navigation algorithm are marked green. These are signals, which are directly necessary for the navigation algorithm. Output signals are marked in orange. Algorithms which use pre-filter based on Kalman filter are marked in gray. Algorithms which use direct Kalman filters are marked in blue. Other blocks in which partial calculations are carried out are marked with yellow or white. The argument B represents the body frame and the argument N represents the navigation frame. Both frames have orientation as NED coordination system.

4. ALGORITHMS WITH KALMAN PRE-FILTER

In Fig. 3, two blocks are using Kalman pre-filters: “KP-IMU&R” and “KP-GPS”. Block “KP-IMU&R” processes two types of data: direct data from the IMU, which are represented by the body acceleration $\mathbf{a}(\mathrm{B})$ and the body angular velocity $\boldsymbol{\omega}(\mathrm{B})$, and data from the magneto-compass (angle yaw γ_{R}) and barometer (h_{MSL} – height above mean sea level). Task of this block is to make Kalman signal pre-filtering. By this pre-filter also the value of angular acceleration $\dfrac{d\boldsymbol{\omega}(\mathrm{B})}{dt}$ is calculated.

Block “KP-GPS” processes data from GPS and strapdown algorithm. This block calculates the following values: h_{MSL}, position and velocity in the navigation frame N, the difference between the height of the reference ellipsoid h_{RE} and h_{MSL}, and the ECEF reference point which defines zero of the navigation frame N. These parameters are calculated using Kalman filtering. The ratio between the covariance matrix of process noise and the covariance matrix of measurement noise is dependent on the total translational acceleration of the body in the navigation frame N $\mathrm{Sd}:\mathbf{a}_{\mathrm{T}}(\mathrm{N})$ determined by strapdown algorithm.

5. ALGORITHMS WITH DIRECT KALMAN FILTER

In Fig. 3, three blocks are applying direct Kalman filters: “DKF-EA”, “DKF-hMSL”, and “DKF-Triplet”.

Block “DKF-EA” processes two types of data:

- data from the IMU as total acceleration $\mathbf{a}(\mathrm{B})$ and angular velocity of rotation $\boldsymbol{\omega}(\mathrm{B})$;

- reference data about the azimuth γ_R from the magneto-compass.

In this block, reference values of Euler angles are computed based on the azimuth from the magneto-compass and acceleration values from the accelerometer. These reference values of Euler angles are used for estimation of the real Euler angles using a direct Kalman filter. The mathematical model of the direct Kalman filter is divided into two parts:

- direct computation of Euler angles using the RM (rotation matrix) with respect to the sequence of rotation roll, pitch, and yaw;

- computation of Euler angles using quaternions.

For smaller values of Euler angles, direct computation using the rotation matrix is considered. For larger values of Euler angles, computing over quaternions is applied.

The direct Kalman filter works as follows: if reference data in the actual sample time interval are not known, the filter performs angular velocity integration, resulting in Euler angles or quaternions. If reference data in the actual sample time interval are known, the filter performs estimation and computed offset of angular velocity. This offset of angular velocity is then subtracted of the angular velocity

$$\boldsymbol{\omega}_\mathbf{O}(\mathrm{B}) = \boldsymbol{\omega}(\mathrm{B}) - \boldsymbol{\omega}_\mathbf{offset}(\mathrm{B}) .$$

Block “DKF-hMSL” processes two types of data:

- height above mean sea level from barometer $h_{\mathrm{MSL}}(\mathrm{baro})$;

- height above mean sea level from GPS $h_{\mathrm{MSL}}(\mathrm{GPS})$

In this case, $h_{\mathrm{MSL}}(\mathrm{GPS})$ is the reference signal. The reason is, that GPS can operate in several modes (for example differential GPS). The accuracy of $h_{\mathrm{MSL}}(\mathrm{GPS})$ is generally dependent on many factors. For this reason auxiliary variables as $3D_{\mathrm{ERR}}$ (3D position error), N_{S} (number of the satellites) and S_{GPS2B} ($h_{\mathrm{MSL}}(\mathrm{baro})$ value is replaced by $h_{\mathrm{MSL}}(\mathrm{GPS})$ value with respect to weight of both values) are used. According to the value of the logical variable S_{GPS2B} the direct Kalman filter can perform the estimation. If the estimation, then the direct Kalman filter carried out determines also the offset $h_{\mathrm{MSL,offset}}$, which is subtracted of the $h_{\mathrm{MSL}}(\mathrm{baro})$. The result $d = h_{\mathrm{MSL}}(\mathrm{baro}) - h_{\mathrm{MSL,offset}}$ is then subsequently subtracted from the reference $h_{\mathrm{MSL,ref}}$. $h_{\mathrm{MSL,ref}}$ corresponds with the ECEF reference point.

The final result from this block is then an estimated value $D = h_{\mathrm{MSL,ref}} - d = h_{\mathrm{MSL,ref}} - h_{\mathrm{MSL}}(\mathrm{baro}) + h_{\mathrm{MSL,offset}}$.

Under certain assumptions, this value of D may replace the D value in navigation frame N computed by GPS. The advantage of this method is that the value of D is known from barometric measurements even if no GPS signal is available. This method divides one position vector in the navigation frame N into two independent parts: part NE and part D.

Block “DKF-Triplet” processes three types of data:

- translational acceleration $\mathbf{a_T}(\mathrm{N})$ computed from strapdown algorithm;

- velocity in navigation frame N $\mathbf{v}(\mathrm{N})$ from GPS;

- position in the navigation frame N obtained as combination of elements NE of the position vector N from GPS with element D from “DKF-hMSL” block.

This block uses a direct Kalman filter as estimator in the following cases:

- in the current sampling period a reference vector NED or only NE of the position in frame N is known, or

- in the current sampling period a reference vector of the translational velocity from GPS $\mathbf{v_T}(\mathrm{N})$ is known, or

- in the current sampling period only a reference value D as one element from position vector in navigation frame N is known.

If in the current sampling period no reference value of the position in frame N as N or E or D, or $\mathbf{v_T}(\mathrm{N})$ is known, the direct Kalman filter works as integrator for translational acceleration $\mathrm{Sd}:\mathbf{a_T}(\mathrm{N})$ from the strapdown algorithm. If the direct Kalman filter works as estimator, then it computes also the acceleration offset $\mathbf{a_{T,offset}}(\mathrm{N})$, with

$$\mathbf{a_{TO}}(\mathrm{N}) = \mathbf{a_T}(\mathrm{N}) - \mathbf{a_{T,offset}}(\mathrm{N}) .$$

6. STRAPDOWN ALGORITHM

The mathematical model of the strapdown algorithm, which is used for airship navigation purpose, expresses the following relationship

$$\mathbf{a}(\mathrm{B}) = \frac{d\boldsymbol{\omega}(\mathrm{B})}{dt} \times \mathbf{R_{IMU}} + \boldsymbol{\omega}(\mathrm{B}) \times \mathbf{v_T}(\mathrm{B}) + \\ + \boldsymbol{\omega}(\mathrm{B}) \times \left(\boldsymbol{\omega}(\mathrm{B}) \times \mathbf{R_{IMU}}\right) - \mathbf{g}(\mathrm{B}) + \mathbf{a_T}(\mathrm{B}) + \mathbf{a_R}(\mathrm{B})$$

where

$\mathbf{a}(\mathrm{B})$ is the measured acceleration vector; $\mathbf{v_T}(\mathrm{B})$ is the translational velocity vector of the body; $\mathbf{R_{IMU}}$ is the IMU position on the body of the turn-point, $\mathbf{g}(\mathrm{B})$ is the gravitation vector in the body frame, $\mathbf{a_T}(\mathrm{B})$ is the translational vector of the body; $\mathbf{a_R}(\mathrm{B})$ is the residual acceleration vector of others forces in the body frame which are not include in the strapdown algorithm.

In this case in the strapdown algorithm is $\mathbf{a_R}(\mathrm{B}) = 0$, because errors of unmodeling acceleration are solving by $\boldsymbol{\omega}_\mathbf{offset}(\mathrm{B})$, $h_{\mathrm{MSL,offset}}$ and $\mathbf{a_{T,offset}}(\mathrm{N})$. The reason why it is possible to accept this assumption is,that all errors caused by this assumption are small in relation to errors caused by the behaviour of the airship body hull.

In case of $\mathbf{a}_{\mathbf{R}}(\mathrm{B}) = 0$, the strapdown equation is the equation of the „Flat Earth Navigator“.

In the strapdown block there is also the computation of $\left(\boldsymbol{\omega}_{\mathbf{O}}(\mathrm{B}) \times \mathbf{R}(\mathrm{GPS})\right)_{\mathrm{N}}$, which defines the transformation to the navigation frame N with $\mathbf{R}(\mathrm{GPS})$ as GPS antenna position on the airship body of the airship turn-point. This part is necessary to subtract of the $\mathbf{v}_{\mathbf{T}}(\mathrm{N})$ obtained from GPS reciever.

Main output from the strapdown algorithm is the translational acceleration in the navigation frame N: $\mathbf{a}_{\mathbf{T}}(\mathrm{N})$.

7. OTHER BLOCKS

In the ‘other’ blocks of the algorithm (see figure 3), auxiliary inputs and characteristic outputs of the navigation algorithm are calculated.

The clocks for magneto-compass, barometer, and GPS are used as auxiliary inputs. The logical value of these signals is ONE if magneto-compass, barometer, or GPS have an actual value in the actual sample time. This is important for decision of the Kalman filtering.

The following parameters are outputs from the navigation algorithm:

- roll, pitch, and yaw angles;
- the angular velocity $\boldsymbol{\omega}_{\mathbf{O}}(\mathrm{B})$ or $\boldsymbol{\omega}_{\mathbf{O}}(\mathrm{N})$, adjusted by the angular velocity offset $\boldsymbol{\omega}_{\mathbf{offset}}(\mathrm{B})$;
- the translational acceleration from the strapdown algorithm $\mathrm{Sd} : \mathbf{a}_{\mathbf{T}}(\mathrm{N})$;
- the translational acceleration $\mathbf{a}_{\mathbf{T}}(\mathrm{N})$ which is result with respect to all measurement units and which is adjusted by offset $\mathbf{a}_{\mathbf{T,offset}}(\mathrm{N})$;
- the translational velocity $\mathbf{v}_{\mathbf{T}}(\mathrm{N})$;
- the position $\mathbf{pos}(\mathrm{N})$;
- the height above mean sea level h_{MSL} ;
- the height above the reference ellipsoid h_{RE} ;
- the ECEF reference point

8. CONCLUSIONS AND RESULTS

To show the results, a comparison of a commercial navigation system with a system that was developed at our department is accomplished. The latter is used as one part this algorithm (the full mathematical model of the navigation algorithm can not be presented here due to limited space). For this comparison we used a commercial navigation system in the price range of 4000 €. For a comparison, 25 journeys by car were made. The mounting of both navigation systems were done in a way that the conditions were as similar as possible as on an airship hull (see figure 4).

Both navigation systems are mounted on a special fixture which is placed in the trunk of a car. This fixture consists of a top platform, a lower platform, and four foam springs. The navigation systems are mounted on the top platform. The lower platform is based in the trunk. When the car is running, the springs cause movements of the top platform contrary to the movement of the car. This is the main objective of the fixture. This mechanism emulates an airship with implemented navigation system. The airship hull is a non-solid body with spring effects. Another reason why the comparison was done on the car is, that at the time of article writing our navigation system did not offer a DGPS mode. Driving a car and matching with Google maps is an alternative strategy.

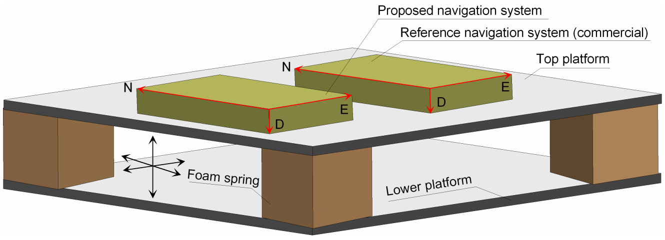

Fig. 4. Fixture of the navigation systems in the car.

Figures 5 and 6 show Google maps with plotted averaged paths (from 25 journeys by car) obtained from the navigation system. The real paths are driven with respect to the traffic rules. In figures 5 and 6, two characteristic points are marked. Point 1 is located at traffic lights. Point 2 is a parking position. From figures 5 and 6, it is easy to see that if the car stopped the navigation algorithm begins "walking". This is caused by the GPS data, which are obtained in normal user mode. If the body is in motion, it is possible to eliminate this “walking” of the GPS by taking into account the orientation of the body (car).

The precision of the proposed navigation system on the defined path is better than the commercial navigation system (4000 €) about from 0,5 to 2,5m.

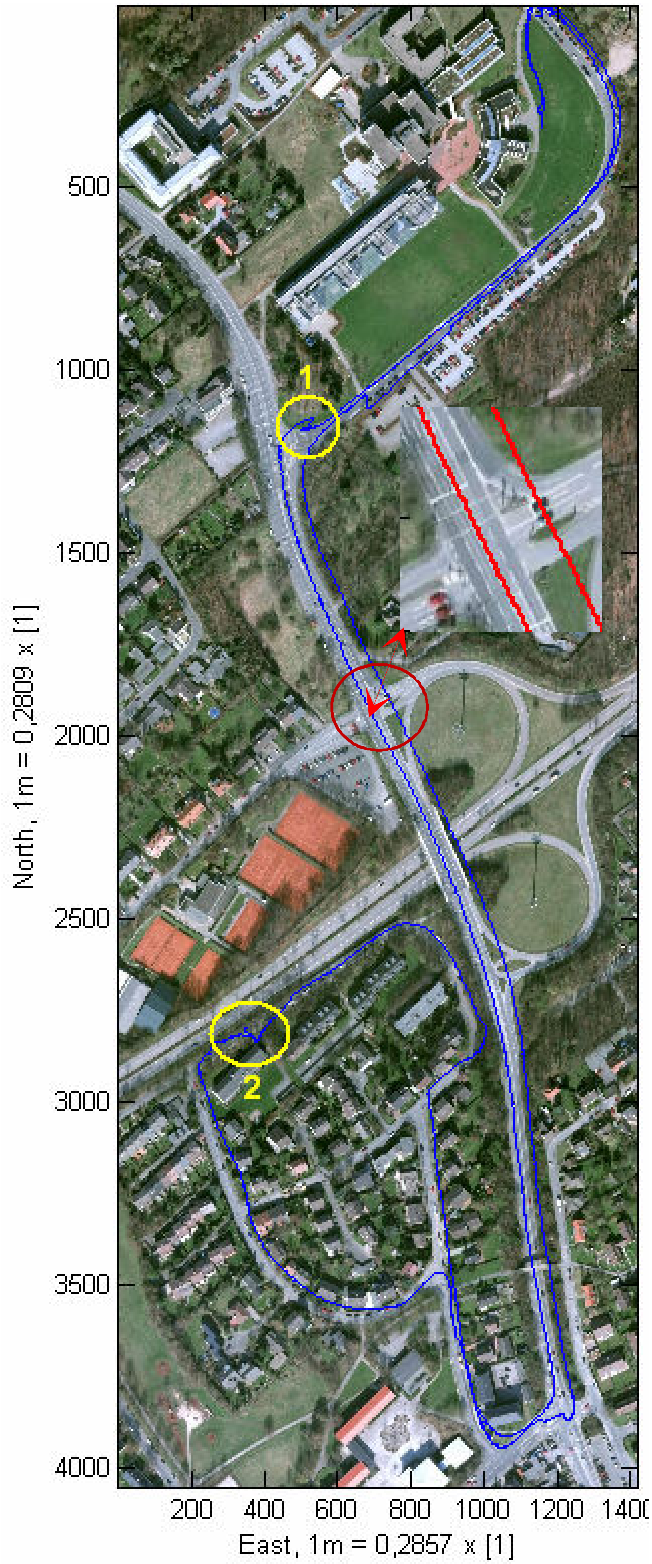

Fig. 5. Reference navigation system (commercial).

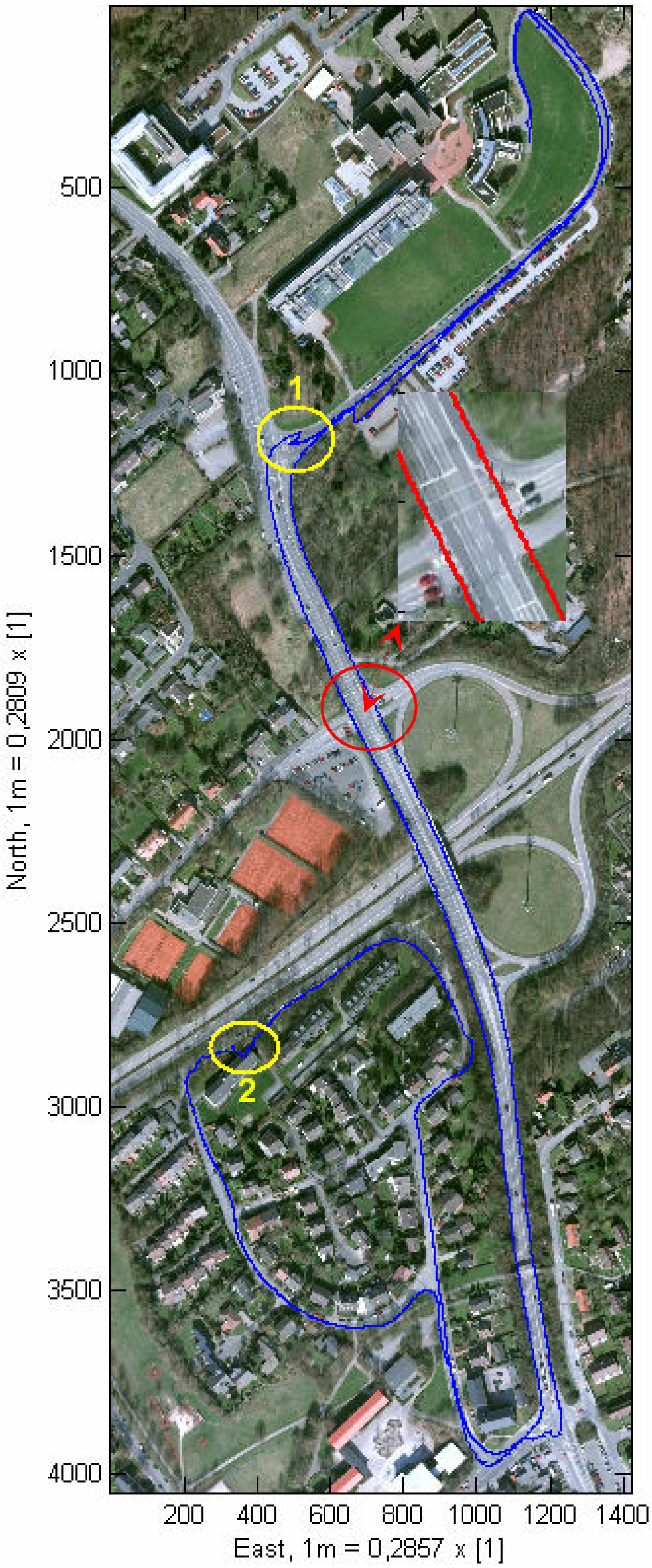

Fig. 6. Proposed navigation system.

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