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Identification of nonlinear systems with general output backlash

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Abstract: The paper deals with identification of nonlinear cascade systems with general output backlash, where instead of the straight lines determining the upward and downward parts of backlash characteristic general curves are considered. A new form of general backlash description is leading to the mathematical model, which has all the model parameters separated. The identification based on this model is solved as a quasi-linear problem using an iterative algorithm with internal variables estimation.

1. INTRODUCTION

One of the most important nonlinearities that limit control systems performance in many applications is the so-called backlash (Kalas et al, 1985). Unfortunately, there are only few contributions in the literature on the identification of systems with backlash (Bai, 2002), (Cerone and Regruto, 2007), (Dong et al, 2009), (Dong et al, 2010), (Giri et al, 2008), (Hägglund, 2007), (Vörös, 2010a), (Vörös, 2010b).

In control systems it is assumed that the backlash is "linear", i.e., straight lines approximate the upward and downward curves of the characteristic (Tao and Kokotovic, 1993), (Tao and Canudas de Wit, 1997), (Nordin and Gutman, 2002). This simplifies the system description, however, in some cases it may lead to inaccuracies. The components of control systems may be free from backlash when new, but after some time in use the wear results in an introduction of backlash in the systems. In general the form of backlash changes with time and wear, regardless of what form of backlash was present when the component was new. Therefore it may be appropriate to generalize the backlash and consider general upward and downward curves. The only works dealing with the identification of systems with general switch and backlash nonlinearities were published in (Giri et al, 2010), (Rochdi et al, 2010a), (Rochdi et al, 2010b). The proposed approach is based on two independent, but structurally symmetric identification schemes. The first one determines the points located on the descendent border of general nonlinearity as well as the parameters of the linear subsystem. The second identification scheme determines the points located on the ascendent border of general nonlinearity and the parameters of the linear subsystem. The key idea is to use pulse-type periodic input signals so that only the points of interest are excited on each border.

In this paper an identification method for cascade systems with output backlash based on a new mathematical model for general backlash is presented. First, an analytic description of this hard dynamic nonlinearity is described, which uses appropriate switching functions and their complements (Vörös, 2009). Then the identification method for cascade systems consisting of a linear dynamic system followed by a general output backlash is proposed. This is based on a mathematical model, where the parameters of linear dynamic system and the parameters characterizing the general backlash are separated, hence their estimation can be solved as a quasi-linear problem using an iterative method with internal variable estimation (Vörös, 2001, 2003, 2007).

2. GENERAL BACKLASH MODEL

In the case of "linear" backlash the left and right branches of the characteristic are considered to be straight lines. However, in some applications the straight lines are only advantageous approximations of general curves constituting the left and right branches of backlash as shown in Fig. 1.

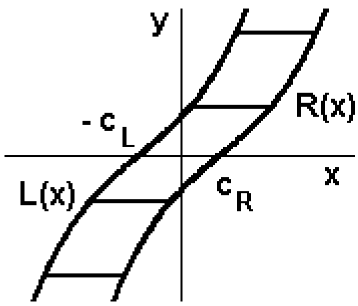

Fig. 1 General backlash characteristic

The general backlash characteristic can be described by the equation (Vörös, 2009)

$$y(t) = \begin{cases} L[x(t)] & x(t) \le z_L \\ y(t-1) & z_L \le x(t) \le z_R \\ R[x(t)] & x(t) \ge z_R \end{cases} \qquad (1)$$

where the mappings *L[x(t)]* and *R[x(t)]* describe the left and right branches of the characteristic, respectively, the x-axis values z_L and z_R are given as follows:

$$y(t-1) = L(z_L) \tag{2}$$

$$y(t-1) = R(z_R) \tag{3}$$

Assume the left and right curves can be approximated by the polynomials

$$L[x(t)] = \sum_{i=1}^{n} m_{Li}[x(t)+c_L]^i \tag{4}$$

$$R[x(t)] = \sum_{i=1}^{n} m_{Ri}[x(t)-c_R]^i \tag{5}$$

respectively, where $c_L > 0$, $c_R > 0$ are the intersections of *L[x(t)]* and *R[x(t)]* with the x-axis. Then the general backlash characteristic can be written as

$$y(t) = \begin{cases} \sum_{i=1}^{n} m_{Li}[x(t)+c_L]^i & x(t) \le z_L \\ y(t-1) & z_L \le x(t) \le z_R \\ \sum_{i=1}^{n} m_{Ri}[x(t)-c_R]^i & x(t) \ge z_R \end{cases} \tag{6}$$

where

$$y(t-1) = \sum_{i=1}^{n} m_{Li}[z_L + c_L]^i \tag{7}$$

$$y(t-1) = \sum_{i=1}^{n} m_{Ri}[z_R - c_R]^i \tag{8}$$

After introducing the internal variables

$$\xi_1(t) = x(t) + c_L \tag{9}$$

$$\xi_2(t) = x(t) - c_R \tag{10}$$

the following variables based on (7) and (8) can be defined:

$$f_1(t) = h[\xi_1(t)] = h[\sum_{i=1}^{n} m_{Li}\xi_1^i(t) - y(t-1)] \tag{11}$$

$$f_2(t) = h[\xi_2(t)] = h[y(t-1) - \sum_{i=1}^{n} m_{Ri}\xi_2^i(t)] \tag{12}$$

where the switching function

$$h(s) = \begin{cases} 0, & if\ s > 0 \\ 1, & if\ s \le 0 \end{cases} \tag{13}$$

is switching between two sets of values, i.e., (− ∞, s) and (s, ∞). Then the general backlash can be modeled by one difference equation as follows:

$$y(t) = \sum_{i=1}^{n} m_{Li}\xi_1^i(t) f_1(t) + \sum_{i=1}^{n} m_{Ri}\xi_2^i(t) f_2(t) + y(t-1)[1-f_1(t)][1-f_2(t)] \tag{14}$$

To include the deadzone parameters c_L and c_R into the backlash model, we can separate the first terms of the sums in (14) and half-substitute from (9) and (10) as follows:

$$y(t) = m_{L1}x(t) f_1(t) + m_{L1}c_L f_1(t) + \sum_{i=2}^{n} m_{Li}\xi_1^i(t) f_1(t) + m_{R1}x(t) f_2(t) + m_{R1}c_R f_2(t) + \sum_{i=2}^{n} m_{Ri}\xi_2^i(t) f_2(t) + y(t-1)[1-f_1(t)][1-f_2(t)]. \tag{15}$$

Now the input/output relation for the generalized backlash (15) is identical with that of (1). All the model parameters are separated and the model is linear in the input, output and internal variables. This model allows the upward and downward curves to be different provided that the intersection of the two curves is not in the region of practical interest.

3. SYSTEMS WITH GENERAL OUTPUT BACKLASH

In many real control systems the backlash appears in a cascade connection with linear dynamic systems. One of the simplest cases is the cascade system where a linear dynamic system is followed by a backlash as shown in Fig. 2. The linear dynamic system can be described by the difference equation as

$$x(t) = \sum_{i=1}^{na} a_i u(t-i) - \sum_{j=1}^{nb} b_j x(t-j) \tag{16}$$

where *u(t)* and *x(t)* are the inputs and outputs, respectively.

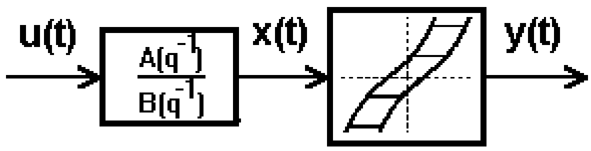

Fig. 2 Cascade system with general output backlash

Let the general backlash be described by (15). The output equation of this cascade system can be constructed by connecting (15) and (16). However, a direct substitution of (16) into (15) would lead to a quite complex expression, therefore the so-called key term separation principle can be applied (Vörös, 2010c). It means that (16) will be substituted only for $x(t)$ in the first term of (15). Moreover, in this connection of two systems we can assume that $m_{L1} = 1$, hence the model equation for the cascade system with general output backlash can be written as

$$\begin{aligned} y(t) &= \sum_{i=1}^{na} a_i u(t-i) f_1(t) - \sum_{j=1}^{nb} b_j x(t-j) f_1(t) + \\ & c_L f_1(t) + \sum_{i=2}^{n} m_{Li} \xi_1^i(t) f_1(t) + m_{R1} x(t) f_2(t) + \\ & m_{R1} c_R f_2(t) + \sum_{i=2}^{n} m_{Ri} \xi_2^i(t) f_2(t) + \\ & y(t-1)[1-f_1(t)][1-f_2(t)] \end{aligned} \tag{17}$$

where the parameters of both the linear system and the general backlash are separated and the equation is quasi-linear as the variables $\xi_1(t)$, $\xi_2(t)$, $f_1(t)$ and $f_2(t)$ depend on the backlash parameters and the internal variable $x(t)$ depends on the linear system parameters. Defining the vector of data

$$\begin{aligned} \varphi(t) &= [u(t-1) f_1(t), \cdots, u(t-na) f_1(t), \\ & -x(t-1) f_1(t), \cdots, -x(t-nb) f_1(t), \\ & f_1(t), \xi_1(t)^2 f_1(t), \cdots, \xi_1(t)^n f_1(t), x(t) f_2(t), \\ & -f_2(t), \xi_2(t)^2 f_2(t), \cdots, \xi_2(t)^n f_2(t)]^T \end{aligned} \tag{18}$$

and the vector of parameters

$$\begin{aligned} \theta &= [a_1, \cdots, a_{na}, b_1, \cdots, b_{nb}\, c_1, m_{L2}, \cdots, m_{Ln}, m_{R1}, \\ & c_2, m_{R2}, \cdots, m_{Rn}]^T \end{aligned} \tag{19}$$

where

$$m_{L1} = 1, \quad c_L = c_1, \quad c_R = c_2 / m_{R1}, \tag{20}$$

the cascade system with general output backlash can be written in the vector form as follows:

$$y(t) - y(t-1)[1-f_1(t)][1-f_2(t)] = \varphi^T(t)\,\theta\ . \tag{21}$$

As the variables $\xi_1(t)$, $\xi_2(t)$, $f_1(t)$, $f_2(t)$ and the internal variable $x(t)$ in (18) are unmeasurable and must be estimated, an iterative parameter estimation process has to be considered similarly as in (Vörös, 2007). Assigning the internal variable $x(t)$ in the s-th step as

$$^s x(t) = \sum_{i=1}^{na} {}^s a_i u(t-i) - \sum_{j=1}^{nb} {}^s b_j\, {}^s x(t-j) \tag{22}$$

and the estimated variables $\xi_1(t)$, $\xi_2(t)$, $f_1(t)$ and $f_2(t)$ in the s-th step as

$$^s\xi_1(t) = {}^s x(t) + {}^s c_L \tag{23}$$

$$^s\xi_2(t) = {}^s x(t) - {}^s c_R \tag{24}$$

$$^s f_1(t) = h[\sum_{i=1}^{n} {}^s m_{Li}\, {}^s\xi_1^i(t) - y(t-1)] \tag{25}$$

$$^s f_2(t) = h[y(t-1) - \sum_{i=1}^{n} {}^s m_{Ri}\, {}^s\xi_2^i(t)] \tag{26}$$

the error to be minimized in the estimation procedure is

$$\begin{aligned} ^{s+1}e(t) &= y(t) - y(t-1)[1-{}^s f_1(t)][1-{}^s f_2(t)] - \\ & {}^s\varphi^T(t)\ {}^{s+1}\theta \end{aligned} \tag{27}$$

where $^s\varphi(t)$ is the data vector with the corresponding estimates of variables $x(t)$, $\xi_1(t)$, $\xi_2(t)$, $f_1(t)$ and $f_2(t)$ according to (22) – (26) and $^{s+1}\theta$ is the (s+1)-th estimate of the parameter vector.

The steps in the iterative procedure may be now stated as follows:

a) Minimizing the least squares criterion based on (27)

$$^{s+1}J = \frac{1}{N} \sum_{t=1}^{N} {}^{s+1}e^2(t) \tag{28}$$

where N is the number of measured input and output samples, the estimates of parameters $^{s+1}\theta$ are computed using $^s\varphi(t)$ with the s-th estimates of variables $^s x(t)$, $^s\xi_1(t)$, $^s\xi_2(t)$, $^s f_1(t)$ and $^s f_2(t)$.

b) Using (22) – (26) the estimates of $^{s+1}\varphi(t)$ are evaluated by means of the recent estimates of corresponding parameters.

c) If the estimation criterion is met the procedure ends, else it continues by repeating steps a) and b).

In the first iteration only the parameters of linear dynamic system are estimated and the initial values can be chosen zero. However, nonzero initial values of the general backlash parameters m_{R1}, c_L and c_R have to be considered for evaluation of $^1\varphi(t)$ to start up the iterative algorithm.

4. CONCLUSIONS

In this paper a new analytic form of general backlash characteristic description was used in the mathematical model for cascade systems including this type of dynamic nonlinearity in the output. Iterative algorithm was proposed enabling simultaneous estimation of both the backlash parameters and the parameters of the cascaded linear dynamic system on the basis of input/output data.

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