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Structure and Parametric Definition of Linear Dynamic Object via Identification Based on Real Interpolation method

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Abstract: The paper considers identification problem of linear dynamic object based on real interpolation method. It is provided the solution of raised problem that gives formalized algorithm for structure and parameters definition using as input data step-response function. Suggested approach gives possibility to build transfer function of analyzed object with pre-determinate level of relevant error in time domain. Numerical example is provided.

Keywords: Identification, transfer function, real interpolation method.

1. INTRODUCTION

Identification problem plays significant role in automation control system design, operation and diagnosis. Mathematical model type and plant parameters determination enables to design and adjust controllers with a high accuracy, to create self-tuning systems etc.

It is convenient to use transfer functions for describing mathematical models of linear object. In this case identification problem consists of two subtasks. The first one is called structure identification and includes mathematical model structure determination, in particular, polynomial degrees determination of transfer function numerator and denominator. The second subtask is parametric identification. It includes transfer function coefficients definition of given structure.

For the case, when the dynamics of an analyzed object could be described with the help of linear differential equations, there exist quite a large number of solutions for identification problem (Hildebrand et al. 2007), (Agüero et al. 2010) and (Liu et al. 2010).

Real interpolation method showed high efficiency in parametric identification problem solution. However there is no formalized and algorithmically efficient solution of structure identification problem within this method. For example, in the work (Rudnitsky et al. 2008), the single structure according to Ishlinsky rule is chosen for all plants. Such approach introduces excessive redundancy in case of objects identification, which is described by low order differential equations, and do not allow reproducing specific characteristics of 4th and higher order objects behavior. In the paper (Antropov et al. 2004) it is offered to identify transfer function structure by enumerative technique, starting from the first order object with the further increase of denominator and numerator polynomial orders. The order is increasing until error in a time domain reaches specified level. Such approach is rather effective, but it is desirable to reduce number of considered variants of transfer function structures.

The main goal of this paper is to suggest more efficient algorithm for structure identification of transfer function such as degree orders of numerator and denominator using step response function as input information. The raised problem requires decreasing number of iterations in case of enumeration of structure parameters such as degree orders of numerator and denominator.

In the paper is considered new approach for structure and parametric identification, which is based on real interpolation method.

2. BASIS OF REAL INTERPOLATION METHOD

It is known that signal images could be obtained on the basis of appropriate real transformation, based on the direct Laplace transform formula

$$F(p) = \int_0^\infty f(t)e^{-pt}dt, \tag{1}$$

where $f(t)$ is an original function, $F(p)$ – Laplace representation of given function, $p = \delta + j\omega$ – complex transformation variable. There are several important things from point of mathematical description and automation control system design. Functions $F(p)$ are images and therefore their usage is more preferable in contrast with time functions $f(t)$. For instance, time function differentiation operation $f(t)$ in the Laplace domain, corresponds to function multiplication operation in case of zero initial conditions $F(p)=L\{f(t)\}$ by variable *p*. Integration in the original domain corresponds to the division of function $F(p)$ by variable p.

Taking into consideration the fact that real interpolation method operates in a real domain, the formula of such transformation could be obtained by replacement of transformation complex variable p for real δ in the expression (1)

$$F(\delta)=\int_0^\infty f(t)e^{-\delta t}dt. \qquad (2)$$

The most important moment when describing automation control system from mathematical point of view while obtaining the result of expression (2) is the lack of imaginary unit. Conditions of function $F(\delta)$ existence and uniqueness are defined by integral convergence (2). Therefore variable δ is constrained:

$$F(\delta)=\int_0^\infty f(t)e^{-\delta t}dt,\ \delta\in[C,\infty),\ C\geq 0. \qquad (3)$$

Expression (3) is a formula of direct integral real transformation or δ-transformation.

In application to linear automation control system, when $f(t)$ represents its dynamic characteristic of plant, convergence (3) is provided by choosing an appropriate parameter *C* value. For stable system with an impulse step response *C* may be assumed as 0.

In literature (Goncharov 1995) this method is known as real interpolation method (RIM)

3. PARAMETRIC IDENTIFICATION OF DYNAMIC OBJECT

To obtain image functions $F(\delta)$ by time functions $f(t)$ it is possible to use the method, which is based on direct δ-transformation formula (3). This method of obtaining mathematical description of signals is convenient to use in case when original function $f(t)$ in analytical form is known or such function could be obtained by means of experimental tabular data interpolation.

In the general case function $W(\delta)$ can be obtained on the basis of transfer function definition, as output signal image $W(\delta)$ to input signal image $X(\delta)$ ration (in case of zero initial conditions)

$$W(\delta)=\frac{Y(\delta)}{X(\delta)}=\int_0^\infty y(t)e^{-\delta t}dt \Big/ \int_0^\infty x(t)e^{-\delta t}dt\,. \qquad (4)$$

The «input-output» correlation takes on a standard form $Y(\delta)=W(\delta)X(\delta)$.

The most suitable form of function approximation on infinite intervals of an argument changing is fractionally rational representation of the following type

$$W(\delta)=\frac{b_m\delta^m+b_{m-1}\delta^{m-1}+...+b_1\delta+b_0}{a_n\delta^n+a_{n-1}\delta^{n-1}+...+a_1\delta+1},\ n\geq m\,. \qquad (5)$$

The possibility of obtaining real function, based on expression (4) creates favorable preconditions for solving practical and research problems while designing automation control systems via RIM. Realization of these facilities using computers demands transition from analytical expressions to numerical sequences, with further transition from one form of representation to another.

For this aims it is offered in RIM to implement continuous functions $F(\delta)$ discretization, performing their following restoration in continuous form with the help of interpolation. For real function $F(\delta)$, $\delta\in[0,\infty)$, in a node system δ_i, $i=1,2,3...$ a set of values $F(\delta_i)$, $i=\overline{1,\eta}$ is formed, where η – numerical characteristic dimension:

$$\{F(\delta_i)\}_\eta=\{F(\delta_1),F(\delta_2),...,F(\delta_\eta)\}\,. \qquad (6)$$

It is necessary to define the disposition interval and distribution law, when choosing the interpolation nodes. In general case this task do not have any exact solution, therefore uniform distribution law of nodes is used very frequently. Coverage of the domain where the most essential changes for function $F(\delta)$ happen is a requirement made to the interval $\delta\in[\delta_1,\delta_\eta]$.

In the general case, when the inequality $m\leq n$ is hold for the polynomials order of a numerator m and denominator n of function (5), then for δ_η node estimation the following equation is considered:

$$F(\delta_\eta)=(0{,}1\div 0{,}2)[F(0)-F(\infty)]+F(\infty)\,. \qquad (7)$$

The solution of equation (7) is found numerically with the iterative procedures.

Nodes position inside the $[\delta_1,\delta_\eta]$ interval for analytical grid is determined in a following way:

$$\delta_i=\delta_1+\frac{\delta_\eta-\delta_1}{\eta-1}(i-1),\ i=\overline{1,\eta}\,. \qquad (8)$$

Then «input-output» equation for automatic control system design with the real images involvement, written in a form

$$Y(\delta)=W(\delta)X(\delta)\,, \qquad (9)$$

could be represented as a correlation between numerical characteristic elements $\{X(\delta_i)\}_\eta$, $\{Y(\delta_i)\}_\eta$, $\{W(\delta_i)\}_\eta$ of input $x(t)$ and output $y(t)$ signals with a transfer function $W(\delta)$:

$$\{Y(\delta_i)\}_\eta = \{W(\delta_i)\}_\eta \{X(\delta_i)\}_\eta .$$

The usage of a numerical representation for the system signal in aggregate with low operation number turns to be a positive characteristic of the latter equation in contrast to analytical representation (9).

Connection between the model in form of numeric characteristic and real transfer function in form (5) is established by means of linear algebraic equations system

$$W(\delta_i) = \frac{b_m \delta_i^m + b_{m-1}\delta_i^{m-1} + ... + b_0}{a_n \delta_i^n + a_{n-1}\delta_i^{n-1} + ... + a_1\delta_i + 1}, \; i = \overline{1,\eta} . \quad (10)$$

The solution of (10) is found in the following form

$$\begin{cases} b_m\delta_1^m + b_{m-1}\delta_1^{m-1} ... + b_0 - a_n\delta_1^n W(\delta_1) - ... - a_1\delta_1 W(\delta_1) = W(\delta_1), \\ b_m\delta_2^m + b_{m-1}\delta_2^{m-1} ... + b_0 - a_n\delta_2^n W(\delta_2) - ... - a_1\delta_2 W(\delta_2) = W(\delta_2), \\ ... \\ b_m\delta_\eta^m + b_{m-1}\delta_\eta^{m-1} ... + b_0 - a_n\delta_\eta^n W(\delta_\eta) - ... - a_1\delta_\eta W(\delta_\eta) = W(\delta_\eta). \end{cases}$$

As a result of linear algebraic equations system solution, unknown transfer function $W(\delta)$ coefficients will be found. And their number will be equal to parameter η. For the case of representation a transfer function in form (5) the value of this parameter is $\eta = n + m + 1$, that provides linear algebraic equation system (10) solution unicity. The transition from real transfer function $W(\delta)$ to Laplace representation is realized by formal substitution of complex transformation variable $p \rightarrow \delta$ in transfer function expression.

The principal unsolved problem at this stage is a choice of the transfer function structure. That is numerical value n and m determination in expression (5). Let's consider this problem deeper.

4. IDENTIFICATION STRUCTURE OF TRANSFER FUNCTION

Transfer function structure problem, that is numerator m and denominator n polynomial order, can be solved by the method offered in paper (Shalaev 2005). Let's use equation (5) and extreme correlation

$$\lim_{\delta\to\infty} \frac{W(\delta)}{W(g \cdot \delta)} = g^{n-m}, \quad (11)$$

Where $g > 1$ is a real number. From the resulting correlation (11) structure parameters estimation is obtained

$$\gamma = \frac{\ln(g^{n-m})}{\ln(g)} = \widehat{n-m} . \quad (12)$$

Real number is obtained with a help of formula (12) that contains integer and fractional parts. Let's assume fractional part as one unit and add to integer part.

Unfortunately, it is rather difficult to define the limit (11) analytically, because the expressions for $W(\delta)$ and $W(g \cdot \delta)$ are defined according to the formula (4). In connection with this it is offered to restrict an interval $W(\delta)$ of essential function changes and consider not extreme correlation (11), but the expression

$$\frac{W(\delta_\eta)}{W(g \cdot \delta_\eta)} \cong g^{n-m} , \quad (13)$$

where node δ_η is defined by solution of the equation (7) with substitution $\delta \rightarrow g \cdot \delta_\eta$, since denominator transfer function (12) changes g times faster, than in numerator. Then magnitude δ_η specifies right interval boundary $[\delta_1, \delta_\eta]$ of interpolation nodes distribution δ_i, $i = \overline{1,\eta}$. Having a single step response and taking into account extreme correlations $\lim_{\delta\to 0} W(\delta) = \lim_{t\to\infty} h(t)$ and $\lim_{\delta\to\infty} W(\delta) = \lim_{t\to 0} h(t)$, it is possible to write in the right part of expression (7)

$$W(g\delta_\eta) = (0{,}1 \div 0{,}2)[h(\infty) - h(0)] + h(0) , \quad (14)$$

where $h(\infty)$, $h(0)$ – final and initial step response values $h(t)$ correspondingly. Then γ evaluation is found as a result of equation (14) and further calculations for expressions (13) and (12). With the help of obtained structure parameters estimation γ it is possible to form the transfer function structure identification algorithm. To make this, it is necessary to express numerator m order through denominator n order and estimation value γ

$$m = n - \gamma , \; n = \begin{cases} 1, 2, ... \text{ if } \gamma = 0, \\ \gamma, \gamma + 1, ... \text{ if } \gamma \neq 0. \end{cases} \quad (15)$$

Value n is a free argument in the last equation. The existence of the fixed parameter γ, allows to get rid of examination a set of transfer function structures, which do not meet the requirements of equation (15). Parameter n value enumeration should be continued until relative identification error satisfies specified criterion in the time domain.

5. NUMERICAL EXAMPLE

Let's examine structure and parametric identification approach. As a result of the experiment, the signal $x(t) = 1(t)$ (with zero initial conditions) was feed to the object input. Object response $y(t) = h_{obj}(t)$ is represented on figure 1.

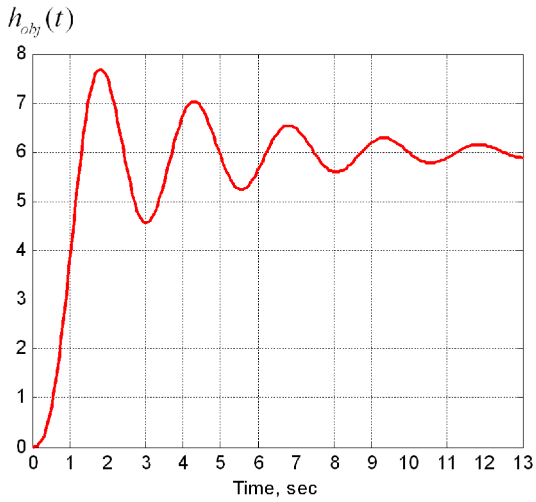

Fig. 1. Step-response of given object

For calculating $W_{obj}(\delta)$ in expression (4) the numerical integration formula is used (Collins 2003)

$$W_{obj}(\delta)=\frac{\left(\frac{h_{obj}(t_0)e^{-\delta t_0}+h_{obj}(t_k)e^{-\delta t_k}}{2}+\sum_{j=1}^{k-1}h_{obj}(t_j)e^{-\delta t_j}\right)\cdot\Delta T}{\left(\frac{1(t_0)e^{-\delta t_0}+1(t_k)e^{-\delta t_k}}{2}+\sum_{j=1}^{k-1}1(t_j)e^{-\delta t_j}\right)\cdot\Delta T},$$

where ΔT – input and output signal sampling period accordingly, t_j = $j{\cdot}\Delta T$ – current moment of time, $j=\overline{0,k}$ $(k=59)$. Let's assume ΔT = 0.22 sec. Notice that experimental data were obtained on the interval $t\ge 0$, so that function $1(t)$ corresponds to the constant value $1(t)$ = 1, in this example. Taking into account simplifications, the latter expression could be represented in the form of

$$W_{obj}(\delta)=\frac{\frac{h_{obj}(t_0)e^{-\delta t_0}+h_{obj}(t_k)e^{-\delta t_k}}{2}+\sum_{j=1}^{k-1}h_{obj}(t_j)e^{-\delta t_j}}{\frac{e^{-\delta t_0}+e^{-\delta t_k}}{2}+\sum_{j=1}^{k-1}e^{-\delta t_j}}.$$

Further let's define interval boundaries, inside which interpolation nodes will be placed. Since object is stable, it's possible to assume δ_1=0 for the left bound. Right bound will be defined by node δ_η placement. Its value will be found from equation (14)

$$W_{obj}(g\cdot\delta_\eta)=0.2\cdot[h_{obj}(t_k)-h_{obj}(t_0)]+h_{obj}(t_0)\,.$$

Let's place real transfer function $W_{obj}(\delta)$ expression in a left side of equation, and then turn to equation examination

$$\frac{\frac{h_{obj}(t_0)e^{-g\delta_\eta t_0}+h_{obj}(t_k)e^{-g\delta_\eta t_k}}{2}+\sum_{j=1}^{k-1}h_{obj}(t_j)e^{-g\delta_\eta t_j}}{\frac{e^{-g\delta_\eta t_0}+e^{-g\delta_\eta t_k}}{2}+\sum_{j=1}^{k-1}e^{-g\delta_\eta t_j}}= \qquad (16)$$
$$=0.2\cdot[h_{obj}(t_k)-h_{obj}(t_0)]+h_{obj}(t_0).$$

After accepting a parameter value as $g=2$ (Shalaev 2005) and using numeric values for the last equation, it is possible to find the solution of the equation (16). In order to solve equation (16) iterative procedure is used. For given numeric values the solution is $\delta_\eta=2.15$ with an accuracy of $|\varepsilon|\le 0{,}01$.

Let's implement substitution (13) to (12) and find structure estimation

$$\gamma=\ln\left(\frac{W_{obj}(\delta_\eta)}{W_{obj}(g\cdot\delta_\eta)}\right)\frac{1}{\ln(g)}=\ln\left(\frac{W_{obj}(2.15)}{W_{obj}(2\cdot 2.15)}\right)\frac{1}{\ln(2)}=1.88.$$

Finally according to recommendation given earlier let's assume $\gamma=2$. Then according to (15) let's write

$$m=n-2,\;\; n=2,3,\dots\,. \qquad (17)$$

The last expression provides information about relative degree of transfer function model $W_m(\delta)$.

Nodes system is formed according to expression (8). Then while taking into account structure parameter estimation linear algebraic equation system (10) is composed. The solution gives possibility for defining transfer function model $W_m(p)$ parameters. As proximity criterion for identified transfer function $W_m(p)$ and object transfer function $W_{obj}(p)$ the estimation in time domain is used in the following form

$$I=\max_{t\in[0,T]}\left(100\frac{|h_{obj}(t)-h_m(t)|}{h_{obj}(t)}\right)\le E\,,$$

where $h_m(t)$ – step response of transfer function $W_m(p)$, $T=13$ sec – observation time for given object, E = 5 % – maximum identification error.

Using expression (16) it is possible to make a conclusion that transfer function identification procedure should be started from the second order model with m = 0 degree of numerator and n = 2 degree of denominator. If the identification error isn't satisfy proximity criterion then more complex structure of transfer function $W_m(\delta)$ should be used. According to expression (17) denominator degree is n = 3 and numerator degree is $m=1$. This iterative process is continuing until identification error will reach a pre-determinate level.

Let's demonstrate how to compose equation system (10) for the transfer function $W_m(\delta)$ with with m = 0 degree of numerator and n = 2 degree of denominator:

$$\begin{cases} W_{obj}(\delta_1) = \dfrac{b_0}{a_2\delta_1^2 + a_1\delta_1 + 1}, \\ W_{obj}(\delta_2) = \dfrac{b_0}{a_2\delta_2^2 + a_1\delta_2 + 1}, \\ W_{obj}(\delta_3) = \dfrac{b_0}{a_2\delta_3^2 + a_1\delta_3 + 1}, \end{cases}$$

where interpolation nodes are performed on the base of formula (8): $\delta_1 = 0$, $\delta_2 = 1.07$, $\delta_3 = 2.15$. The solution is: $b_0 = 5.9$, $a_2 = 0.63$, $a_1 = 0.47$. The value of proximity criterion is $I = 72.3$ %. Given solution doesn't satisfy pre-determinate level of error E. It means that structure of transfer function $W_m(\delta)$ should be more complex.

The results of identification for different structures of transfer function $W_m(\delta)$ are presented in table 1.

Table 1. Identification results

Degree	Transfer function $W_m(p)$	I, %
$m = 0$, $n = 2$	$\dfrac{5.986}{0.629p^2 + 0.469p + 1}$	72.3
$m = 1$, $n = 3$	$\dfrac{-0.547p + 5.986}{0.036p^3 + 0.279p^2 + 0.597p + 1}$	27.7
$m = 2$, $n = 4$	$\dfrac{0.068p^2 + 4.695p + 5.986}{0.107p^4 + 0.292p^3 + 0.909p^2 + 1.543p + 1}$	4.93

After analyzing the results we could see that appropriate approximation was reached with the proximity criterion value $I = 4.93$ %, for transfer function

$$W_m(p) = \frac{0.068p^2 + 4.695p + 5.986}{0.107p^4 + 0.292p^3 + 0.909p^2 + 1.543p + 1}.$$

On the figure 2 step-response functions for this model $h_m(t)$ and object $h_{obj}(t)$ are represented.

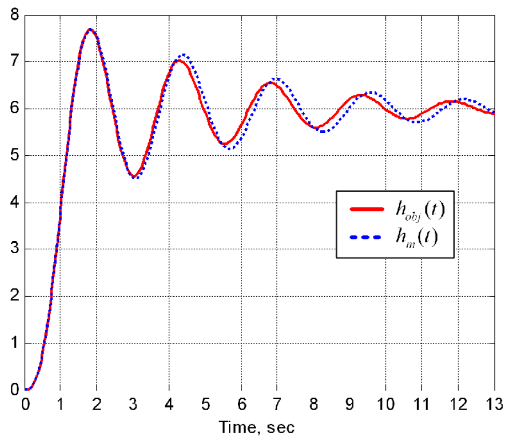

Fig. 2. Step-responses of object and identified model

Thus in case of simple enumeration we have to consider transfer functions with following combinations of numerator and denominator degrees: (m = 0, n = 2), (m = 1, n = 2), (m = 2, n = 2), (m = 0, n = 3), (m = 1, n = 3), (m = 2, n = 3), (m = 3, n = 3), (m = 0, n = 4), (m = 1, n = 4) and (m = 2, n = 4). Instead of considering ten different variant of transfer functions for parameters estimation it was used only three structures, which were determinate according to expression (16). This obviously demonstrates that proposed identification procedure provides rather efficient way of model estimation for linear dynamic object.

6. CONCLUSION

This paper presents a new method of structure and parametric identification of linear dynamic systems based on RIM which was improved with more efficient algorithm of structure definition. The efficiency of the method was shown on numerical example. The proposed identification procedure operates with signals in time domain and uses step-response function as input information for further processing. Whereas traditional identification methods (as recursive least squares method) provides only parametric approximation for given function template, the new identification procedure in addition to parameters definition gives possibility of model structure estimation.

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