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Fault Accommodation in Nonlinear Time Delay Systems

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Abstract: Solution to the problem of fault accommodation in nonlinear time delay dynamic systems is related to constructing the control law which provides full decoupling with respect to fault effects. Existing conditions are formulated and calculating relations are given for the control law.

1. INTRODUCTION

Fault tolerant control (FTC) is a tool intended for increasing a reliability and safety for critical purpose control systems. The goal of FTC is to determine such control law that preserves the main performances of the faulty system while the minor performances may degrade. There are two principal approaches to FTC. The first of them involves adaptive control techniques and assumes on-line fault detection and estimation followed by control law accommodation, see, e.g. (Blanke *et al* 2003, Jang *et al* 2006, Staroswiecki *et al* 2006). The second approach is focused on such control law determination which provides full decoupling with respect to fault effects in the output space of the system. In contrast to the first approach, the second one does not need in fault estimation. Therefore, such approach looks reasonable if on-line fault estimation is impossible.

Full decoupling problem solution under appropriate its statement has been obtained in (Isidori 1995) for affine systems. But the possibility of this solution applying in the framework of FTC problem is strictly limited by the demand on the system state vector availability (this vector is immediately included into the control law description). As a rule, not all components of the state vector are immediately measurable in practice, and estimation of full state vector for the system with unknown (affected by the faults) dynamics is impossible.

In (Shumsky *et al* 2009, Shumsky & Zhirabok 2010) a solution to the accommodation problem in nonlinear systems has been obtained on the basis of algebra of functions and differential geometry (Shumsky & Zhirabok 2006). In present paper, this problem is solved for time delay systems. These systems form an important class of nonlinear systems. They are used to represent a wide variety of processes and systems including hydraulic/pneumatic systems, communication systems, biological systems, *etc*. To solve the problem of fault accommodation for this class of systems, we use so-called logic-dynamic approach which allows obtaining a solution for time delay nonlinear systems with no differentiable nonlinearities using linear methods. Besides, we use more sophisticated treatment in contrast to the paper (Shumsky *et al* 2009) which allows obtaining in some cases more simple solution with the point of view of designed systems dimensions.

Consider nonlinear systems described by equation

$$\begin{aligned}\dot{x}(t) &= Fx(t) + F_d x(t-\tau) + C\varphi(Ax(t), u(t)) + \\ &\quad Gu(t) + L\vartheta(t), \\ y(t) &= Hx(t).\end{aligned} \tag{1}$$

In (1), x, y, and u are vectors of state, output, and control; F, F_d, C, A, G, L, and H are known matrices of appropriate dimensions; φ is an arbitrary scalar nonlinear function, $\vartheta(t) \in R^v$ is the vector describing the fault. Assume that for healthy system it holds $\vartheta(t) = 0$. For simplicity, the system with a single nonlinearity is considered. Denote system (1) as Σ.

It is assumed that fault detection procedure is performed by known methods (Blanke *et al* 2003). If a fault occurs, $\vartheta(t)$ becomes an unknown function, and a solution to the control problem based on model (1) becomes impossible. To overcome this difficulty, it is suggested to obtain the vector $u(t)$ according to

$$u(t) = g(y(t), x_0(t), u_*(t)) \tag{2}$$

for some function g where $u_*(t) \in R^m$ is a new control vector, $x_0(t) \in R^q$, $q \le n$, is a state vector of the system has to be determined and described by equation

$$\begin{aligned}\dot{x}_0(t) &= F_0 x_0(t) + F_{d0} x_0(t-\tau) + \\ &C_0\varphi(A_0\begin{pmatrix} x_0(t) \\ y(t)\end{pmatrix}, u(t)) + G_0 u(t) + J_0 y(t).\end{aligned} \tag{3}$$

Note that model (3) does not depend on the unknown vector $\vartheta(t)$.

Assume that the model obtained by substitution (2) into (1) can be transformed to the form

$$\dot{x}_*(t) = F_* x_*(t) + F_{d*} x_*(t-\tau) + C_* \varphi(A_* x_*(t), u_*(t)) + G_* u_*(t) \tag{4}$$

with $x_*(t) \in R^p$, $p \le q$. If the control (2) exists and the fault occurred and detected, then a solution to the control problem is performed on the basis of model (4) which does not contain the unknown vector $\vartheta(t)$. As a result, fault accommodation effect is achieved. Scheme for system Σ control is shown in Figure 1.

Note that the use of the control (2) assumes moving system (1) only in some subspace of its state space which corresponds to the state space of system (4). Under this, the goal of control should be achieved by appropriate choosing the trajectory belonging to this subspace. The need of appropriate trajectory existence (or a possibility to correct the goal of control for finding appropriate trajectory) restricts the sphere of the considered approach application.

The problem is to determine the existing condition for the control (2) and to obtain all matrices describing systems (3) and (4). To solve this problem, is it necessary initially to design the auxiliary system Σ' described by equation

$$\dot{x}'(t) = f'(x'(t), y(t), u(t)) = F'x'(t) + F'_d x'(t-\tau) + C'\varphi(A'\begin{pmatrix} x'(t) \\ y(t) \end{pmatrix}, u(t)) + G'u(t) + J'y(t). \tag{5}$$

2. LOGIC-DYNAMIC APPROACH

So-called logic-dynamic approach developed in (Zhirabok & Usoltsev 2002) will be used for designing the system Σ' . The feature of this approach is the use of conventional linear algebraic tools in contrast to nonlinear algebraic and differential geometric tools of the work (Shumsky & Zhirabok 2006).

The logic-dynamic approach for systems in the form (1) includes the following three steps.

Step 1. Replacing the initial nonlinear system (1) by certain linear system.

Step 2. Solving the problem under consideration for this linear system with some additional restrictions.

Step 3. Transforming the obtained linear system into the nonlinear one by adding a nonlinear term.

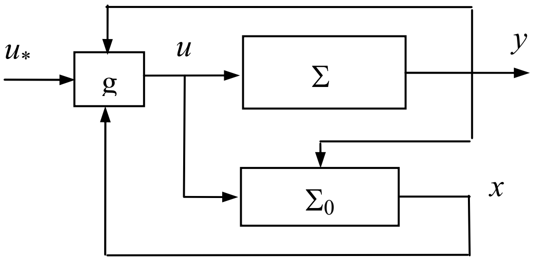

Fig. 1. Scheme for system Σ control

At the first step of this approach, the nonlinear term $C\varphi(Ax(t), u(t))$ is removed from system (1). The corresponding linear system is of the form

$$\dot{x}(t) = Fx(t) + F_d x(t-\tau) + Gu(t) + L\vartheta(t), \quad y(t) = Hx(t). \tag{6}$$

It will be named the linear part of system (1).

At the second step, according to the logic-dynamic approach, a linear part of system (5) is designed. It is well-known from the fault detection and isolation theory of linear systems (Frank 1990) that for this linear part design, the state x' is a linear combination of system (6) state according to

$$\Phi x(t) = x'(t)$$

in the unfaulty case after the response to unlike conditions has died out. We will say, with this equality in mind, that system (5) estimates the initial system state vector with accuracy to a function realized by the matrix Φ. In the absence of faults, the following set of equations can be obtained by analogy with (Zhirabok & Usoltsev 2002):

$$\Phi F = F'\Phi + J'H, \quad \Phi F_d = F'_d \Phi, \quad G' = \Phi G. \tag{7}$$

It follows immediately from definition of the matrix Φ and (7) that

$$\Phi C\varphi(Ax, u) = C'\varphi(A'\begin{pmatrix} x'(t) \\ y(t) \end{pmatrix}, u(t)) = C'\varphi(A'\begin{pmatrix} \Phi x(t) \\ Hx(t) \end{pmatrix}, u(t)).$$

This equality is true if the following relations hold:

$$C' = \Phi C, \qquad A = A'\begin{pmatrix} \Phi \\ H \end{pmatrix}.$$

One can show that the relation $A = A'\begin{pmatrix} \Phi \\ H \end{pmatrix}$ is equivalent to the equality

$$rank\begin{pmatrix} \Phi \\ H \end{pmatrix} = rank\begin{pmatrix} \Phi \\ H \\ A \end{pmatrix}. \tag{8}$$

By analogy, it can be shown that the second equation in (7) is equivalent to equality

$$rank(\Phi) = rank\begin{pmatrix} \Phi \\ \Phi F_d \end{pmatrix}. \tag{9}$$

These conditions are those mentioned at Step 2. If the matrix Φ satisfies the first equation in (7) and these conditions, the problem under consideration can be solved.

To ensure if system (3) is independent of the unknown vector $\vartheta(t)$, or if the full decoupling demand is fulfilled, the equality $\Phi L = 0$ has to hold.

3. SYSTEM Σ' DESIGN

The matrix Φ can be obtained as follows. Introduce the matrix L^0 of maximal row rank such that $L^0 L = 0$. Condition $\Phi L = 0$ implies the equality

$$\Phi = NL^0$$

for some matrix N. Replace the matrix Φ in the first equation in (7) with NL^0 that gives $NL^0 F = F'NL^0 + J'H$ and transform it as follows:

$$(N \quad -F'N \quad -J') \cdot \begin{pmatrix} L^0 F \\ L^0 \\ H \end{pmatrix} = 0. \tag{10}$$

Expression (10) can be considered as an algebraic equation for the matrices N, F', and J'.

Let the matrix $(A \;\; B \;\; C)$ presents all linearly independent solutions to equation (10), i.e.

$$(A \quad B \quad C) \cdot \begin{pmatrix} L^0 F \\ L^0 \\ H \end{pmatrix} = 0. \tag{11}$$

To use the matrices A and B for the system Σ' design, the relation $B = -F'A$ for the matrix F' must hold according to (10) and (11). To obtain these matrices, find rows of the matrix B which are independent of the matrix A rows and remove them from $(A \;\; B \;\; C)$. Denote the obtained matrix as $(A^0 \;\; B^0 \;\; C^0)$. Set $N = A^0$ and $\Phi = NL^0$. If the matrix Φ satisfies conditions (8) and (9), the system Σ' can be built otherwise the problem under consideration is not solvable because in this case full decoupling can not be achieved.

Suppose that conditions (8) and (9) hold. Take $G' = \Phi G$ and $J' = -C^0$; the matrices F' and F'_d are solutions to the algebraic equations $F'N = -B^0$ and $\Phi F_d = F'_d \Phi$ respectively.

As a result, a linear part of the system Σ' is described by the following equation:

$$\dot{x}'(t) = F'x'(t) + F'_d x'(t-\tau) + G'u(t) + J'y(t). \tag{12}$$

At the third step of design, it is necessary to transform the obtained linear system into the nonlinear one. According to (Zhirabok & Usoltsev 2002), the nonlinear term

$$C'\varphi(A'\begin{pmatrix} x'(t) \\ y(t) \end{pmatrix}, u(t))$$

with the matrices $C' = \Phi C$ and A' obtained from the algebraic equation

$$A = A'\begin{pmatrix} \Phi \\ H \end{pmatrix}$$

must be added to the right-hand side of equation (12) that gives

$$\begin{aligned} \dot{x}'(t) &= f'(x'(t), y(t), u(t)) = F'x'(t) + F'_d x'(t-\tau) + \\ & C'\varphi(A'\begin{pmatrix} x'(t) \\ y(t) \end{pmatrix}, u(t)) + G'u(t) + J'y(t). \end{aligned} \tag{13}$$

Note that the algebraic equations $A = A'\begin{pmatrix} \Phi \\ H \end{pmatrix}$ and $\Phi F_d = F'_d \Phi$ are solvable because conditions (8) and (9) hold respectively.

4. CONTROL LAW DETERMINATION

To carry out an analysis of the system Σ', introduce the matrices H' and R whose rows present all linearly independent solutions to the algebraic equation

$$(H' \quad -R)\begin{pmatrix} \Phi \\ H \end{pmatrix} = 0.$$

Note that the vector Ry presents those components of the vector y and their linear combinations which can be computed as a function of the state vector x', i.e. $Ry = H'x'$. Consider two cases.

(1) Every component of the function f' contains only those components of the vector y which depend on the vector Ry components; in this case set

$$u_j = u_{*j}, \qquad j = 1, \ldots, m.$$

This means that a block "g" in Figure1 is absent.

(2) Suppose that Case 1 does not hold and find in the function f' all terms with minimal numbers of variables in the form

$$\alpha_i(x', y, u), \qquad i = 1, \ldots, r,$$

which contain the control u and components of the vector y functionally independent of the vector Ry (some terms do not contain the variable x'). Denote

$$\begin{aligned} u_{*1} &= \alpha_1(x', y, u), \\ &\vdots \\ u_{*r} &= \alpha_r(x', y, u). \end{aligned} \tag{14}$$

To check solvability of these nonlinear algebraic equations, assume that

$$rank\left(\frac{\partial\alpha}{\partial u}\right) = s$$

for all x', y, and u except perhaps on a set of measure zero, where $\alpha = (\alpha_1 \;\; \dots \;\; \alpha_r)^{\mathrm{T}}$, and the function α contains m', $m' \le m$, components of the vector u as its arguments. It is obvious that the inequalities $m' \ge s$ and $r \ge s$ hold by definition of m', r, and s. Assume for simplicity that if some u_j is contained in the function α, then it is not in other part of the function f'. Consider three cases.

(1) $m' = r = s$; in this case the system of equations (14) is solvable for some m' components of the control vector u (without loss of generality suppose that they are $u_1, \dots, u_{m'}$):

$$u_j = \gamma_j(x', y, u_*), \quad j = 1, \dots, m'. \tag{15}$$

Take

$$u_j = u_{*j}, \quad j = m'+1, \dots, m.$$

(2) $m' > r = s$; in this case the function α contains $m' - r$ redundant components of the vector u. Without loss of generality assume that these components are the last $m' - r$ ones, i.e. $u_{r+1}, \dots, u_{m'}$. Using additional equations for these components

$$u_j = u_{*j}, \quad j = r+1, \dots, m, \tag{16}$$

one can solve the system of equations (14) in the form (15) for $j = 1, \dots, r$.

(3) $m' \ge r > s$ or $r > m' \ge s$; in these cases find the matrix P with s rows such that

$$rank\left(P\frac{\partial\alpha}{\partial u}\right) = s$$

for all x', y, and u except perhaps on a set of measure zero. The matrix P collects s functionally independent components from all ones of the function α. The redundant components $u_{s+1}, \dots, u_{m'}$ (when $m' > s$) are now in the function $P\alpha$. Using (16) for $j = s+1, \dots, m$, one can solve the equation

$$u_* = P\alpha$$

in the form (15) for $j = 1, \dots, s$.

5. SYSTEM Σ_0 DESIGN

Note that in some cases all relations in (15) do not depend on the components of the vector x'; in this case the system Σ_0 is absent. In some cases these relations depend on all components of the vector x'; in this case the system Σ_0 coincides with Σ'.

Generally, (15) depends on some components of the vector x', in this case it is necessary to design the nontrivial system Σ_0. To do this, define the matrix Q (by analogy with Φ) such that

$$Qx'(t) = x_0(t) \qquad \forall t. \tag{17}$$

It can be shown that the following set of equations

$$\begin{pmatrix} F_0 \\ F_{d0} \end{pmatrix} Q = \begin{pmatrix} QF' \\ QF'_d \end{pmatrix}, \tag{18}$$

$$A_{01}Q = A'_1 \tag{19}$$

holds where $A' = (A'_1 \;\; A'_2)$, $A_0 = (A_{01} \;\; A_{02})$. Equations (18) and (19) are equivalent to equations

$$rank(Q) = rank\begin{pmatrix} Q \\ QF' \\ QF'_d \end{pmatrix}, \tag{20}$$

$$rank(Q) = rank\begin{pmatrix} Q \\ A'_1 \end{pmatrix} \tag{21}$$

respectively. The matrices F_0, F_{d0}, and A_{01} are obtained from (18) and (19) respectively, other matrices described the system Σ_0 can be found as follows:

$$J_0 = QJ', \quad G_0 = QG',$$
$$C_0 = QC', \quad A_{02} = A'_2. \tag{22}$$

Equality (17) is used for replacing the vector x' in (15) by x_0. As a result, (15) is transformed into

$$u_j = g_j(x_0, y, u_*), \qquad j = 1, \dots, m',$$

corresponding to the general law (2).

The matrix Q can be constructed according to the following procedure. Let $x'^{(1)}$ be a subvector of x' whose components are in the function α and $x'^{(1)} = Q^{(1)}x'$. Consider three cases.

(1) If conditions (20) and (21) hold with $Q = Q^{(1)}$, then set $Q = Q^{(1)}$ and define the matrices F_0, F_{d0}, J_0, G_0, C_0, A_0 from (18), (19), and (22) respectively.

(2) If some rows of the matrix A'_1 (denote them A'') do not satisfy condition (21), set $Q^{(2)} = \begin{pmatrix} Q^{(1)} \\ A'' \end{pmatrix}$, otherwise $Q^{(2)} = Q^{(1)}$. If condition (20) holds with $Q = Q^{(2)}$, then set $Q = Q^{(2)}$ and define the matrices F_0, F_{d0}, J_0, G_0, C_0, A_0 from (18), (19), and (22) respectively.

(3) If some rows of the matrix $\begin{pmatrix} Q^{(2)}F' \\ Q^{(2)}F'_d \end{pmatrix}$ (denote them $Q'^{(2)}$) do not depend on the rows of the matrix $Q^{(2)}$, then set $Q^{(3)} = \begin{pmatrix} Q^{(2)} \\ Q'^{(2)} \end{pmatrix}$ and check condition (20). If it holds, set $Q = Q^{(3)}$ and define the matrices F_0, A_{01}, J_0, G_0, C_0, A_{02} from (18), (19), and (22) respectively. Otherwise repeat above operations until condition (20) satisfies. Let Q be equal to the final matrix $Q^{(*)}$; define the matrices F_0, F_{d0}, J_0, G_0, C_0, A_0 from (18), (19), and (22) respectively.

As a result, the system Σ_0 is described as follows:

$$\dot{x}_0(t) = F_0 x_0(t) + F_{d0} x_0(t-\tau) + C_0 \varphi(A_0 \begin{pmatrix} x_0(t) \\ y(t) \end{pmatrix}, u(t)) + G_0 u(t) + J_0 y(t).$$

6. SYSTEM Σ_* DESIGN

Consider two kinds of components of the function f': the first kind contains components of the vector y functionally independent of the vector Ry and do not depend on the control u; the second kind contains those components of the function α which are not in the function $P\alpha$ (if $P\alpha \neq \alpha$). Denote a set of these components numbers by $N = (n_1, n_2, ..., n_k)$.

If $N \neq \varnothing$, then the system Σ' contains components of the vector y which can not be decoupled from the unknown function $\vartheta(t)$. In this case Σ' must be redesigned as follows. Remove rows with numbers $n_1, n_2, ..., n_k$ from the matrix $(A^0 \;\; B^0 \;\; C^0)$ and analyze the obtained matrix by analogy with the matrix $(A \;\; B \;\; C)$. Denote the obtained matrix as $(A^{00} \;\; B^{00} \;\; C^{00})$ and set

$$N = A^{00}, \quad \Phi = NL^0, \quad G' = \Phi G, \quad J' = -C^{00};$$

the matrix F' is a solution to the equation $F'N = -B^{00}$ (we denote the redesigned system and its elements as the initial ones for simplicity). Other matrices of the redesigned system can be obtained as it is described above.

If $N = \varnothing$, then the system Σ' is not need to be redesigned. Assume that the general description of the initial or redesigned system is given by (13).

Consider the redesigned system Σ' and find all terms in the form

$$\alpha_i(x', y, u), \quad i = 1,2,...,r,$$

which are investigated in Section 4. Replace all these terms by components of the new control vector u_* according to (14). The system Σ' may contain components of the vector y which depend on the vector Ry only. These components must be replaced by components of the vector x' as follows. Suppose that some y_j is in Σ' and $y_j = \delta(Ry)$ for some function δ. Then

$$y_j = \delta(Ry) = \delta(RHx) = \delta(H'\Phi x) = \delta(H'x').$$

Take

$$x_{*j} = x'_j, \; j = 1, ..., p = \dim x'.$$

These replacements transform the system Σ' into the system Σ_*.

7. ILLUSTRATIVE EXAMPLE

Consider the system described by the model

$$\dot{x}(t) = \begin{pmatrix} -x_1(t) - x_1(t)x_5(t) + u_2(t) \\ x_1(t-\tau) + u_1(t) - x_1(t) - x_2(t) - \vartheta(t) \\ x_4(t) - x_1(t) - x_2(t) + u_1(t) - \vartheta(t) \\ x_3(t) - x_2(t) - x_4(t) \\ x_1(t) + x_2(t) + \vartheta(t) \end{pmatrix},$$

$$y(t) = \begin{pmatrix} x_1(t) \\ x_4(t) \\ x_5(t) \end{pmatrix}.$$

The following matrices can be chosen for logic-dynamic description of the initial system:

$$F = \begin{pmatrix} -1 & 0 & 0 & 0 & 0 \\ -1 & -1 & 0 & 0 & 0 \\ -1 & -1 & 0 & 1 & 0 \\ 0 & -1 & 1 & -1 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{pmatrix}, \quad F_d = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$C = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad G = \begin{pmatrix} 0 & 1 \\ 1 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}, \quad L = \begin{pmatrix} 0 \\ -1 \\ -1 \\ 0 \\ 1 \end{pmatrix},$$

$$A^{(1)} = (1 \;\; 0 \;\; 0 \;\; 0 \;\; 0), \quad A^{(2)} = (0 \;\; 0 \;\; 0 \;\; 0 \;\; 1),$$
$$\varphi(A^{(1)}x, A^{(2)}x, u) = x_1 x_5.$$

The matrix L^0 is computed as follows:

$$L^0 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}.$$

Equation (11) $(A \quad B \quad C)\begin{pmatrix} L^0F \\ L^0 \\ H \end{pmatrix} = 0$ has several independent solutions for matrices A^0, B^0, and C^0, one of them is

$$(A^0 \quad B^0 \quad C^0) =$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & -1 & 1 & 0 & 0 & 0 \end{pmatrix}.$$

Therefore

$$N = A^0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad \Phi = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix},$$

$$F' = \begin{pmatrix} -1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & -1 \end{pmatrix}, \quad J' = -C^0 = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$G' = \Phi G = \begin{pmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}, \quad C' = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}.$$

It can be shown that

$$F'_d = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

$$A'^{(1)} = (1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0),$$

$$A'^{(2)} = (0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1).$$

It is easily to check that conditions (8) and (9) hold. As a result, system (3) is described as follows

$$\dot{x}'(t) = F'x'(t) + F'_d x'(t-\tau) + J'y(t) + C'\varphi(A'x(t), u(t)) =$$
$$\begin{pmatrix} -x'_1(t) + x'_1(t)y_3(t) + u_2(t) \\ x'_1(t-\tau) + u_1(t) \\ y_2(t) + u_1(t) \\ -x'_2(t) + x'_3(t) - y_2(t) \end{pmatrix}.$$

An analysis shows that

$$R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \quad Ry = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}, \quad N = \varnothing .$$

Because the first equation contains the variable y_3 which is functionally independent of Ry, then take $\alpha(x', y, u) = x'_1 y_3 + u_2$, $r = m' = s = 1$. Set $u_{*2} = x'_1 y_3 + u_2$, then $u_2 = u_{*2} - x'_1 y_3$. It is easy to show that $Q^{(1)} = (1 \quad 0 \quad 0 \quad 0)$. One can check that conditions (20) and (21) hold, then $Q = Q^{(1)}$, $x_{01} = x'_1$, and the system Σ_0 is described by equation

$$\dot{x}_{01}(t) = -x_{01}(t) - x_{01}(t)y_3(t) + u_2(t).$$

Finally, law (2) takes a form

$$u_1 = u_{*1}, \qquad u_2 = u_{*2} - x_{01}y_3. \tag{23}$$

Since $N = \varnothing$, the system Σ' is not need to be redesigned. Description of the system Σ_* is as follows:

$$\dot{x}_* = \begin{pmatrix} -x_{*1}(t) + u_{*2}(t) \\ x_{*1}(t-\tau) + u_{*1}(t) \\ y_2(t) + u_{*1}(t) \\ -x_{*2}(t) + x_{*3}(t) - y_2(t) \end{pmatrix}. \tag{24}$$

It should be noted that methods suggested in the paper (Shumsky *et al* 2009) can be modified for time delay systems. Applying them to the considered example, one obtains the following results: the system Σ_0 coincides with Σ' and is 4-dimensional in contrast to the above 1-dimensional system Σ_0; an equation for the control u_1 is not trivial in contrast to our case $u_1 = u_{*1}$; the system Σ_* is 2-dimensional in contrast to our 4-dimensional system (24).

For simulation, set $\tau = 1$, the control $u_1(t) = \sin t$, $u_2(t) = 5\sin t$. The function $\vartheta(t)$ is modelled by variate with the mean equal to zero and the variance equal to 20. Figure 2 shows the output y_1 behaviour under $\vartheta(t) = 0$; Figure 3 shows the output y_1 behaviour under $\vartheta(t) \neq 0$ without use of the law (2); Figure 4 shows the output y_1 behaviour under $\vartheta(t) = 0$ with use of the law (23).

Clearly, this law provides full decoupling the output y_1 with respect to the fault, and the fault accommodation effect has been achieved.

8. CONCLUSION

The problem of fault accommodation in nonlinear time delay systems has been studied. More general case with several nonlinearities can be considered based on the logic-dynamic approach by analogy with (Zhirabok & Usoltsev 2002). Since this approach uses linear operations only, it is easy to show that the theory described in the paper can be applied to discrete-time dynamic systems.

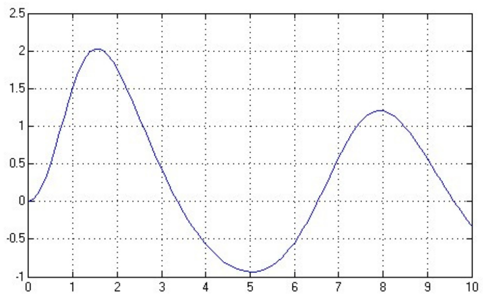

Fig. 2. Output y_1 behaviour under $\vartheta(t) = 0$

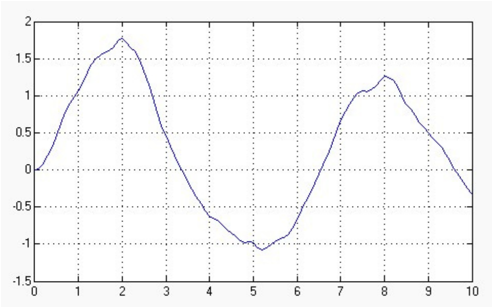

Fig. 3. Output y_1 behaviour under $\vartheta(t) \neq 0$ without correction of the input u_2

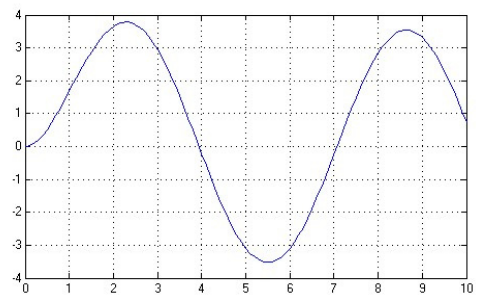

Fig. 4. Output y_1 behaviour under $\vartheta(t) \neq 0$ with the input u_2 corrected according (23)

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