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COMPACT IDENTIFICATION OF PROCESS STATIC GAIN AND ONE POINT OF FREQUENCY RESPONSE

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Abstract: The aim of this paper is to present a new process identification method suitable for automatic tuning of PID controllers. The method combines two experiments together. The first experiment is a process static gain identification and the second is a relay experiment which identifies one point of the process frequency response. Moreover a constant-phase filter is used in the relay feedback loop to get a frequency response sample with phase shift different from −180 degrees. The constant-phase filter parameters are tuned during the first experiment part.

Keywords: Relay, static gain identification, process model, constant-phase filter.

1. INTRODUCTION

The classical control theory says that process controllers can be designed on the base of a few points of the process frequency response. In this direction, the limit method is the popular Ziegler-Nichols frequency method (Ziegler and Nichols, 1942) which uses only one so called ultimate point. The relay identification experiment became one of the most popular in process control. Although the consequential tuning method is not very reliable (Schlegel, 2002) the tradition to identify a frequency response sample with phase shift -180 degrees survives over the decades (Luyben, 1987; C.C.Hang et al., 2002; Huang and Chen, 1996).

The processes that are consistent with the information given by one point of frequency response generate a wide set of models. It is necessary for controller design to reduce this set. One way is to add more points of the process frequency response (Yu, 1999), (Li et al., 1991), (Leva, 1993), (Tan et al., 1999). Extended identification methods are based on enhanced or repeated relay test. A bias relay, a cascade relay, a parasitic relay and other relay modifications can be included in this identification methods group. The big disadvantage of these methods are their time consuming experiments. Other wide identification methods group is based on Fast Fourier Transformation (FFT) (Wang et al., 2003). It is shown in (Mertl and Schlegel, 2007) that the knowledge of two points completed with some additional a priory assumptions on monotonicity is sufficient for accurate design of PID controller. Moreover the second point can be a process static gain in the special case.

This paper presents a new identification method completely different from above mentioned ones. The method combines two experiments together. The first experiment is a process static gain identification using pulse or step input signal. The second is a relay experiment which identifies one point of process frequency response. Moreover a constant-phase filter is used in the relay feedback loop to get a frequency response sample with phase shift different from −180 degrees. The constant-phase filter parameters are tuned after the first identification experiment part. The presented identification method is designed according to the compact autotuning controller needs.

The paper is organized as follows: In Section 2, some basic properties of relay feedback and a constant phase filter based on reference fractional-order integrator are reminded. Section 3 describes the new static gain identification method. Illustrative examples are given in Section 4. Section 5 contains concluding remarks and ideas for future work.

2. ONE POINT OF PROCESS FREQUENCY RESPONSE IDENTIFICATION

2.1 *Relay feedback*

Relay feedback has attracted significant research attention for more than century. The classical work of Tsypkin (Tsipkyn, 1984) on analysis summarizes the progress till 1960s. In 1950s, relays were mainly used as amplifiers but such applications are obsolete now, because there is the big development of electronic technology. In 1960s, relay feedback was applied to adaptive Control. In 1980s Åström and Hägglund introduced their well known work in which they successfully applied the relay feedback method to auto-tune PID controllers for process control. Since then many researchers have come up with novel results and new tools have been developed.

The main advantages of the relay feedback are:

(1) It is a closed-loop test. Therefore the process will not drift away from the operating point.
(2) For the processes with a long time constant it is a more time-efficient method than conventional step or pulse testing.
(3) It identifies the process around the important frequency, for ideal relay at the ultimate frequency with the phase shift -180^o. When we use an appropriate filtr, we can change the phase shift to another more proper value, e.g. for PID controller -135^o.

The relay feedback system is the feedback loop with an ideal (on-off) relay, see Fig. 2 without the adaptive filtr. Consider a relay of magnitude h is inserted in the feedback loop. Initially, the process input $u(t)$ is increased by h. As the output $y(t)$ starts to increase, the relay switches to the opposite value $u(t) = -h$. The close-loop system may start to oscillate with the period P_U and the phase lag is -180^o. The period P_U of the limit cycle is the ultimate period. The ultimate frequency ω_U is given by

$$\omega_U = \frac{2\pi}{P_U}. \quad (1)$$

Using the Fourier series expansion, the periodic $u(t)$ can be written as

$$u(t) = \frac{4h}{\pi}\sum_{n=0}^{\infty}\frac{\sin((2n+1)\omega t)}{2n+1}. \quad (2)$$

The amplitude a of the process output $y(t)$ can be considered to be the result of the primary harmonic of the relay output. Therefore, the ultimate gain can be approximated as (Åström and Hägglund, 1984)

$$K_U = \frac{4h}{\pi a}, \quad (3)$$

where h is the height of the relay and a is the amplitude of oscillation. With ideal relay, a small amount of noise can make the relay switch randomly. By introducing hysteresis, the noise must be larger than the hysteresis width to make the relay switch. For the relay with hysteresis, the frequency response point corresponding to the ultimate gain is transformed to another one which can be approximated as

$$G(\mathrm{j}\omega) = -\frac{\pi\left(\sqrt{a^2-\epsilon^2} - \mathrm{j}\epsilon\right)}{4h}, \quad (4)$$

where h is the height of the relay, a is the amplitude of oscillation and ϵ is the relay hysteresis width.

Further, we use this technique to compute the amplitude of the process frequency response point. The phase will be determined by a constant-phase filter inserted in the loop.

2.2 *Constant-phase filter*

The relay identification of sample with arbitrary phase shift can based on relation

$$\arg P(\mathrm{j}\omega_U) + \arg F(\mathrm{j}\omega_U) = -180 \quad [\text{deg}], \quad (5)$$

where $P(s)$ is the process, $F(s)$ is the suitable filter added to the relay feedback and ω_U is the frequency of steady closed loop oscillations.

One way to obtain the frequency sample with arbitrary phase shift is to complete the relay feedback by a special adaptive low-order filter. The filter parameters are changed during the relay experiment in order to reach the proper phase shift. The adaptation makes the experiment very time consuming namely for slow temperature or chemical processes.

It follows out from (5), that the filter $F(s)$ with constant phase shift in the entire frequency band will speed up identification significantly. For simplicity, the phase shift of the filter must be determined by one user parameter. Fractional-order integro-differential operator described by irrational transfer function

$$F(s) = \frac{1}{s^m}, \qquad m \in \mathbf{R} \tag{6}$$

fulfills our requirements. Frequency response of (6) can be computed by substituting $s = \mathrm{j}\omega$ as

$$\begin{aligned} F(\mathrm{j}\omega) &= \frac{1}{(\mathrm{j}\omega)^m} = \frac{1}{\omega^m}(-\mathrm{j})^m = \\ &= \frac{1}{\omega^m}\left(\cos\frac{\pi}{2}m - \mathrm{j}\sin\frac{\pi}{2}m\right). \end{aligned} \tag{7}$$

Hence, the frequency response in complex plane is a straight line going through the origin and the constant-phase property is ensured. Assuming (5) and (7) the parameter m is related to required phase shift φ as

$$m = 2 - \varphi/90. \tag{8}$$

It is impossible to realize the transfer function (6) precisely in the entire frequency band by finite dimensional filter. However, if the frequency band of the filter is restricted to the limited interval (e.g. 3 decades) then it is possible to design low order constant-phase filter (Mertl, 2008).

Example 1. Let us choose the following example to illustrate the proposed method. Consider the transfer function of the second order plus dead time system

$$P(s) = \frac{1}{(15s+1)(s+1)} \; e^{-0.5s}. \tag{9}$$

We design for each phase lag $\varphi_1 = 135^o, \varphi_2 = 160^o$ and $\varphi_3 = 200^o$ one constant-phase filter (CPF) on 3 decades. The proper values of the central frequency ω_C are $\omega_{C1} = 13.85$, $\omega_{C2} = 22.28$ and $\omega_{C3} = 31.46$, respectively (Mertl J., 2008). The phase lags φ_1, φ_2 and φ_3 correspond to the CPF powers (6) $m_1 = 0.5, m_2 = 0.2222$ and $m_3 = -0.2222$.

The CPF transfer functions designed by the method presented in (Čech M. et al., 2008) are given by

$$F_1(s) = \frac{7.285\ 10^{-6}s^4 + 0.006125s^3 + 0.4134s^2 + 3.402s + 3.108}{0.0003158s^4 + 0.06627s^3 + 1.544s^2 + 4.385s + 1},$$

$$F_2(s) = \frac{0.000002035s^4 + 0.001665s^3 + 0.1331s^2 + 1.321s + 1.417}{0.00001148s^4 + 0.005311s^3 + 0.2657s^2 + 1.649s + 1},$$

$$F_3(s) = \frac{0.000002201s^4 + 0.001437s^3 + 0.1015s^2 + 0.89s + 0.762}{0.0000003612s^4 + 0.0004175s^3 + 0.04713s^2 + 0.6601s + 1}.$$

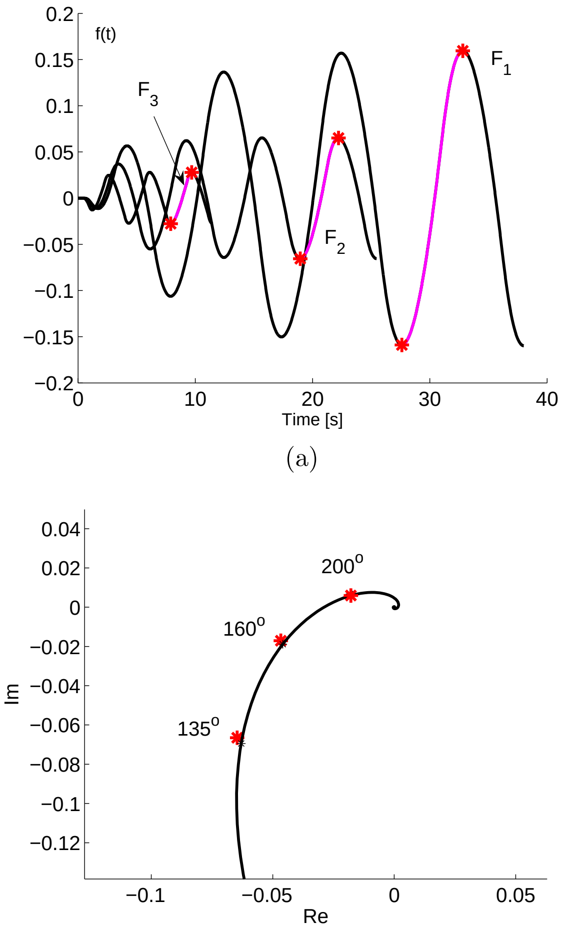


Fig. 1. (a) The filter output oscilations, (b) process Nyquist plot with computed points (black star) and identified points (red star).

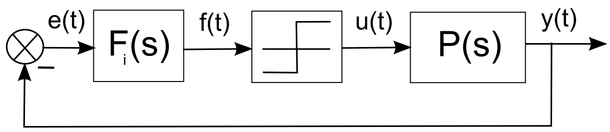


Fig. 2. The relay feedback simulation scheme.

The relay loop is shown in Figure 2. The three relay experiments were done with filters $F_1(s)$, $F_2(s)$ and $F_3(s)$. The resulting oscillations from the CPF output $f(t)$ are depicted in Figure 1 (a). The identification is stopped, when the variance in amplitude between two peaks is less then 1%. The peaks used to compute the amplitude a are marked by the red star in Figure 1 (a). The relay amplitude is set to $h = 1$. The ultimate gain of the system $P(s)$ with the CPF can be computed using equation (4). For the frequency points $G(\mathrm{j}\omega_U)$ it holds

$$\begin{aligned} G_i\,(\mathrm{j}\omega_{Ui}) &= F_i(\mathrm{j}\omega_{Ui})P(\mathrm{j}\omega_{Ui}) = \\ &= -\left(\frac{\pi}{4h}\sqrt{a_i^2 - \epsilon^2} - \mathrm{j}\frac{\pi\epsilon}{4h}\right), \quad i = 1,2,3. \end{aligned} \tag{10}$$

Further, the resulting frequency response points depicted in Fig. 1 (b) are computed as

Table 1. The results of the illustrative example.

	Amplitude	Phase shift [deg]	ω_U [rad/s]
φ_1	0.09277863	134.1801	0.6047
φ_2	0.04967579	159.9205	0.9652
φ_3	0.01937714	200.333	1.7551

$$P(\mathrm{j}\omega_{Ui}) = \frac{G_i(\mathrm{j}\omega_{Ui})}{F_i(\mathrm{j}\omega_{Ui})},\ i = 1, 2, 3. \qquad (11)$$

The results are summarized in the table 1.

3. PROCESS STATIC GAIN IDENTIFICATION

The process static gain identification method is based on three new process characteristic numbers (process order independent).

3.1 *Process characteristic numbers*

Let us assume that the process is described by first three moments m_1, m_2, m_3 of the impulse response $h(t)$ defined as

$$m_i = \int_{t=0}^{\infty} t^i h(t)\mathrm{d}t, \qquad i = 0, 1, 2 \qquad (12)$$

or equivalently by more suitable group of numbers κ, μ, σ^2 defined as follows

$$\kappa = m_0,\ \mu = \frac{m_1}{m_0},\ \sigma^2 = \frac{m_2}{m_0} - \frac{m_1^2}{m_0^2}. \qquad (13)$$

The numbers κ, μ, σ^2 have the required interpretation as they define the process gain, time scale and normalized dead time, respectively. Further it is possible to restrict ourselves to normalized values for gain and time, thus $\kappa = 1, \mu = 1$. Then, process dynamics varies from first-order to pure dead time depending on one parameter σ^2.

3.2 *Static gain identification*

Identification of static gain from a classical relay experiment is not possible. There exist different modifications of relay test to identify the process static gain, e.g using unsymmetrical relay. These methods are often unreliable and the identification results are not sufficiently precise for the controller parameters design.

Therefore we decided to extend the standard relay experiment with the second identification experiment to get the static gain. Two methods are suitable from our point of view:

(1) static gain is computed using separate identification method before the relay test

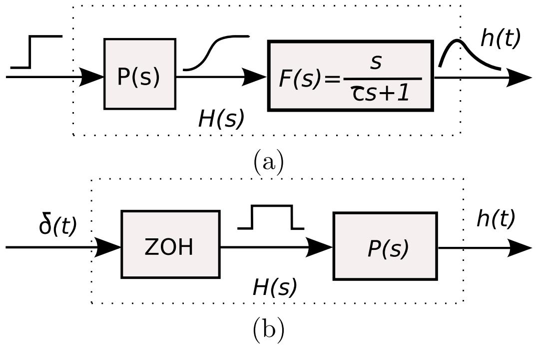


Fig. 3. (a) The step experiment, (b) the pulse experiment.

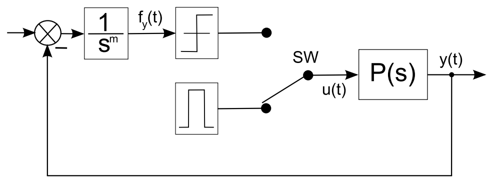


Fig. 4. Scheme of whole identification experiment - pulse static gain identification and relay identification of one frequency characteristic point

(2) static gain is computed from the starting phase of the process – when the process reaches some steady state working point (e.g. starting from cold state of a temperature process).

3.2.1. *Pulse identification experiment*

Assume the process is in steady state. The testing input signal is a pulse with defined amplitude. The pulse length is derived from the percentage change of the process output. It means, that the input pulse is finished when the process output has been changed more then user-defined percentage, see Fig. 5(a). The output is measured till the response damp out. The first three moments m_1, m_2, m_3 can be computed from this measured impulse response. The three process characteristic numbers κ, μ and σ^2 are obtained by (13).

The impulse response $h(t)$ is the system response to the input Dirac pulse $\delta(t)$. Assume the serial connection $H(s)$ of a zero-order hold (ZOH)

$$T(s) = \frac{1}{s}\left(1 - \mathrm{e}^{-Ls}\right) \qquad (14)$$

and the identified process $P(s)$ in Fig. 3(b). The impulse response of the system $H(s)$ is identically the same as the response of the system $P(s)$ to the rectangular unit pulse of the length L. Consequently, the characteristic numbers $\kappa_H, \mu_H, \sigma_H^2$ of the model $H(s)$ can be computed from the response of the system $P(s)$ to the rectangular pulse. For the characteristic numbers κ, μ, σ^2 of the process $P(s)$ it holds (Schlegel et al., 2002)

$$\kappa = \kappa_H / L,$$

$$\mu = \mu_H - L/2,$$
$$\sigma^2 = \sigma_H^2 - L^2/12. \qquad (15)$$

It is not necessary to measure the process response till the steady state. The measurement is possible to stop, when the response exponentially declines. The unmeasured part is then extrapolated by proper exponential function. This advancement makes the identification experiment shorter. The Example 2 illustrates the presented pulse static gain identification method.

3.2.2. *Step identification experiment* The second type of static gain identification experiment utilizes the starting movement to a working point. Primary it is intended for cases, when the process moves from an idle state to a working point. The identification scheme is in Fig. 3(a). Assume the serial connection $H(s)$ of the identified process $P(s)$ and the first order derivative filter $F(s) = s/(\tau s + 1)$. The step response $h(t)$ of the system $H(s)$ is identically the same as the impulse response of the serial connection of the process $P(s)$ and the filter $F(s)$. For the characteristic numbers κ, μ, σ^2 of the process $P(s)$ it holds (Schlegel et al., 2002)

$$\kappa = \kappa_H, \quad \mu = \mu_H - \tau, \quad \sigma^2 = \sigma_H^2 - \tau^2. \qquad (16)$$

4. EXAMPLES

Let us choose the following examples to illustrate the proposed method. Both these examples integrate the static gain identification and one point of the frequency response identification to one compact identification experiment.

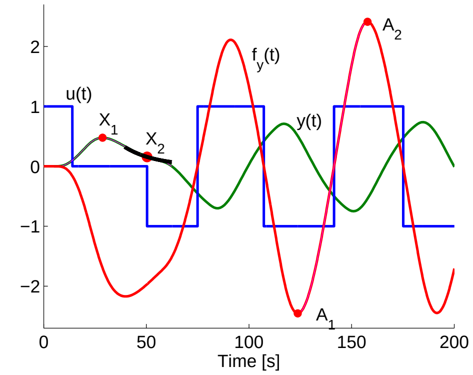
Example 2. (The pulse experiment)
Consider the process transfer function

$$P(s) = \frac{1}{(s+1)(6s+1)(8s+1)^2}\, \mathrm{e}^{-5s}. \qquad (17)$$

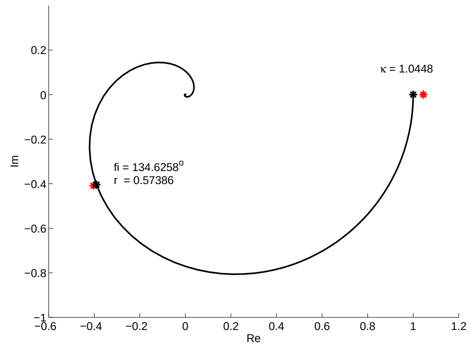
The identification experiment scheme is depicted in Fig. 4. The experiment starts with the selected input pulse amplitude. When the system output $y(t)$ overgrows the given threshold, the pulse is terminated. The maximum X_1 is measured (Fig. 5(a)). The impulse response measuring is stopped, when the output $y(t)$ achieves the point X_2. The rest of the impulse response is extrapolated by proper exponential function (the black curve in Fig. 5(a)). The computed characteristic numbers are

$$\kappa = 1.0448, \;\; \mu = 29.8748, \;\; \sigma^2 = 250.8933. \; (18)$$

In time of the point X_2 the loop is closed with a constant-phase filter and a relay (the switch



(a)



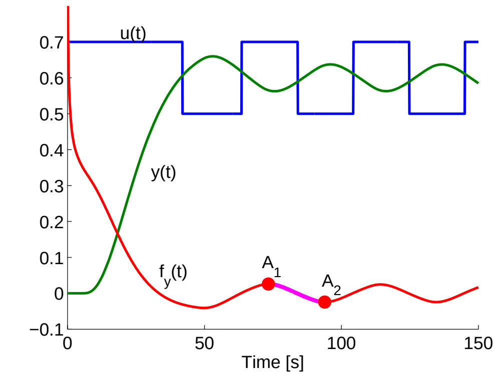
(b)

Fig. 5. (a) Pulse and Relay output $u(t)$, system response $y(t)$ and filter output $f_y(t)$ with marked points A_1, A_2, (b) Identified points (red star) a computed points (black star).

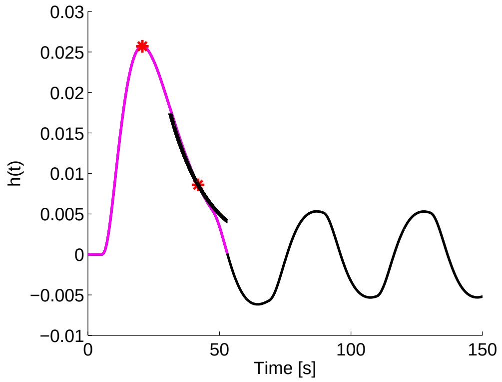
SW in Fig. 4). The central frequency ω_{fc} of the constant-phase filter is set depending on the characteristic numbers μ and σ^2 (Čech M. et al., 2008), which are computed from the pulse part of experiment. The central frequency is $\omega_{fc} = 5.9$. The exact procedure of finding the central frequency ω_{fc} can be found in (Mertl, 2008). The resulting frequency response point and its frequency are computed analogously to Example 1 from the points A_1 and A_2. The results are

$$P(\mathrm{j}\omega_c) = -0.4031 - 0.4084j,$$
$$\omega_c = 0.0927. \qquad (19)$$

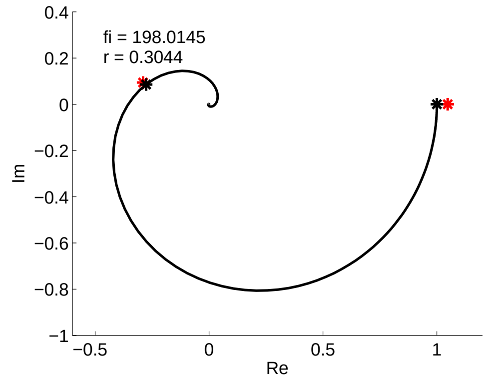
Example 3. (The step experiment)
The process model $P(s)$ is the same one (17) as in the previous example. The identification experiment phases are clear from the Fig. 6(a). The experiment starts from zero steady state and goes to the working point $y(t) = 0.6$. The output $h(t)$ of derivative filter $F(s)$ is depicted in Fig. 6(b). As in the Example 2 the two points X_1 and X_2 are measured and the rest of the impulse response is extrapolated by proper exponential function (the black curve in Fig. 6(b)). Then the process characteristic numbers κ, μ a σ^2 are computed as



(a)



(b)



(c)

Fig. 6. (a) Pulse and Relay output $u(t)$, system response $y(t)$ and filter output $f_y(t)$ with marked points A_1, A_2, (b) Identified points (red star) a computed points (black star).

$$\kappa = 1.0476, \quad \mu = 29.9116, \quad \sigma^2 = 250.1974. \tag{20}$$

When the process reaches the setpoint value, the standard relay experiment starts. The values A_1, A_2 are measured and using describing function method the frequency response point is computed. The constat-phase filter is tuned to identify the point with the phase lag 200^o (Fig. 6(c)). The identification results are

$$P(\mathrm{j}\omega_c) = -0.2895 + 0.0941j, \\ \omega_c = 0.1537. \tag{21}$$

5. CONCLUSION

In this paper an advanced identification method was presented. The method combines two experiments together. The first experiment is a process static gain identification and the second is a relay experiment which identifies one point of the process frequency response. Moreover a constant-phase filter is used in the relay feedback loop to get a frequency response sample with phase shift different from -180 degrees. The constant-phase filter parameters are tuned during the first experiment part. Two different static gain identification methods were presented. Finally, the illustrative examples were shown. The authors believe that the proposed technique can be useful in advanced PID controllers.

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