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CONTROL OF SYSTEMS WITH TIME-VARYING DELAY: AN ALGEBRAIC APPROACH VS. MODIFIED SMITH PREDICTORS

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Abstract: The main aim of this paper is to compare three different control design methods which are applied to a continuous-time single-input single-output (SISO) system with harmonically time-varying delay. The first technique uses a modified PI-PD Smith predictor in combination with standard forms for minimum of integral squared time error (ISTE). The second methodology is also based on modified Smith predictor and on design by Coefficient Diagram Method (CDM). And finally, the third approach to synthesis is grounded in general solutions of Diophantine equations in the ring of proper and Hurwitz-stable rational functions (R_{PS}) for 1DOF or 2DOF control system. The comparison of methods is performed and illustrated on a simulation example.

Keywords: Time-Varying Systems, Time Delay, Modified Smith Predictor, Algebraic Synthesis.

1 INTRODUCTION

Systems affected by time delay (TD) have attracted attention of control theory researchers for decades. The ground of this interest can be seen in common presence of TD in real controlled processes and thus in the necessity of quality and easily applicable control algorithms for this class of systems. Unfortunately, TD always means worse control conditions and, moreover, the situation is much more complicated if it is time-varying.

The possible effective and economical solution for systems with relatively small or limited changes of TD is the usage of robust enough fixed controllers. The worthwhile closed-loop configuration for compensation of dead time has been well known as Smith predictor since 1959. Recently, many new modifications of Smith predictor with improved properties have been introduced – e. g. (Hamamci *et al.* 2001), (Kaya and Atherton 1999), (Majhi and Atherton 1998). Another way how to overcome TD resides in combination of its approximation and subsequent utilization of an algebraic control design method. The advantageous solution represents fractional approach developed in (Vidyasagar 1985), (Kučera 1993) and applied for robust control of TD systems e.g. in (Prokop and Mészáros 1996).

This contribution is focused on control of single-input single-output (SISO) systems with periodically time-varying TD. The results given by continuous-time controller designed in the ring of proper and stable rational functions (R_{PS}) (Prokop and Mészáros 1996), (Prokop and Corriou 1997), (Prokop *et al.* 2002) are compared with those obtained with the use of modified PI-PD Smith predictor (Kaya and Atherton 1999) and also using the Smith predictor designed by Coefficient Diagram Method (CDM) (Hamamci *et al.* 2001).

2 DESCRIPTION OF CONTROLLED SYSTEM

A first order system with time-varying delay described by differential equation:

$$\begin{aligned} y'(t) + y(t) &= u(t - \Theta(t)) \\ \Theta(t) &= 1 + 0.5\sin(5t); \quad y(0) = 0 \end{aligned} \tag{1}$$

is supposed as really controlled plant. As can be seen, TD in (1) is harmonically time-varying from 0.5 to 1.5. As an alternative notation, it can be used also the non-standard hybrid "transfer function" which depends both on complex variable *s* and on time *t*:

$$G(s,t) = \frac{1}{s+1} e^{-[1+0.5\sin(5t)]s} \qquad (2)$$

A mathematical model of controlled system (for the purpose of control design) is represented by time-invariant transfer function:

$$G(s) = \frac{1}{s+1} e^{-s} \qquad (3)$$

for all compared techniques.

3 THE FIRST METHOD: MODIFIED PI-PD SMITH PREDICTOR

The modification of the classical Smith predictor presented in (Kaya and Atherton 1999) comes from the structure with three controllers shown in fig. 1, where G_{c1} is a PI controller, G_{c2} is a PD (or only P where it is appropriate) controller and G_{c3} is the disturbance controller introduced in (Matausek and Micic 1996).

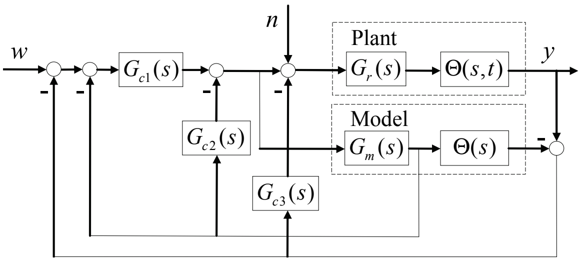

Fig. 1. The modified Smith predictor structure (PI-PD)

Generally, the synthesis is based on usage of standard forms for obtaining the optimal closed-loop transfer function parameters in the meaning of integral squared time error (ISTE) criterion. A concise example of controller computation is shown in the Section 6.

4 THE SECOND METHOD: MODIFIED SMITH PREDICTOR DESIGN BY CDM

The controller design using the Coefficient Diagram Method (CDM) was proposed in (Hamamci *et al.* 2001). This method uses the improved Smith predictor structure with the trio of controllers according to fig. 2.

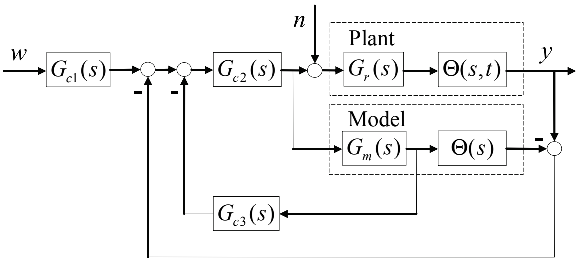

Fig. 2. The modified Smith predictor structure (CDM)

The CDM design is based on the four studies (Coefficient diagram; Modification of Kessler standard form; Lipatov stability analysis; Obtaining characteristic polynomial). Again, a brief illustration of controller calculation is shown in the Section 6. For details about the technique see (Hamamci *et al.* 2001) or related literature.

5 THE THIRD METHOD: ALGEBRAIC CONTROL DESIGN IN R_{PS}

The general two-degree-of-freedom (2DOF) control configuration used in this approach is very well known and shown in fig. 3. The traditional one-degree-of-freedom (1DOF) system can be obtained simply by $R = Q$.

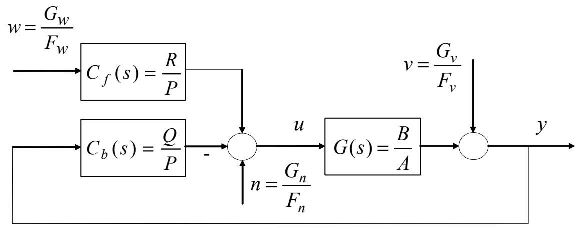

Fig. 3. Two-degree-of-freedom closed loop system

The first step of algebraic control design in R_{PS} for TD systems is to approximate TD member in order that the model becomes usable for linear Diophantine equations. A classical and very suitable tool is Padé approximation. Then it is necessary to describe the systems in R_{PS} as a ratio of two rational fractions:

$$G(s) = \frac{b(s)}{a(s)} = \frac{\frac{b(s)}{(s+m)^n}}{\frac{a(s)}{(s+m)^n}} = \frac{B(s)}{A(s)} \qquad (4)$$

where $n = \max\{\deg(a), \deg(b)\}$ and $m > 0$.

The basic task is now to ensure internal stability of the system in fig. 3. All stabilizing feedback controllers are given by all solutions of the linear Diophantine equation:

$$AP + BQ = 1 \tag{5}$$

with a general solution $P = P_0 + BT$, $Q = Q_0 - AT$, where T is free in R_{PS} and P_0, Q_0 is a pair of particular solutions (Youla – Kučera parameterization of all stabilizing controllers). For details and proofs see (Vidyasagar 1985), (Kučera 1993). The analysis of the control error via condition of divisibility leads to the outcome that for asymptotic tracking F_w must divide AP (or only P in this case) for 1DOF. One of main advantages of the proposed technique is that controllers can be tuned by the only scalar parameter m.

The details, results and references for 2DOF configuration or for other control problems (disturbance rejection, disturbance attenuation, etc.) can be found e. g. in (Prokop and Mészáros 1996), (Prokop and Corriou 1997), (Prokop *et al.* 2002).

6 CALCULATIONS OF CONTROLLERS AND SIMULATION RESULTS

Remind that a controlled plant with time-varying delay is given by (1) or (2) and mathematical model is supposed in the form (3). The controllers for all PI-PD, CDM and R_{PS} design were experimentally tuned to obtain visually acceptable results without or with only small overshot and short settling time. For better comparability, responses with nearly the same time of reaching the reference value were chosen. Furthermore, the following simulation conditions were used: simulation time $T_S = 60s$, reference value 1 with step to 2 in $1/3$ of T_S, load disturbance injected into the plant input $n = -0.6$ in $2/3$ of T_S and no disturbance in the plant output $v = 0$.

For the first method, modified PI-PD Smith predictor, the controlled system model (without TD) has been supposed in the form:

$$G_m(s) = \frac{\beta_0}{s + \alpha_0} = \frac{1}{s+1} \tag{6}$$

The transfer functions of the three controllers are:

$$\begin{aligned} G_{c1}(s) &= K_c\left(1 + \frac{1}{T_i s}\right) = 0.03\left(1 + \frac{1}{0.11s}\right) \\ G_{c2}(s) &= K_f = -0.3299 \\ G_{c3}(s) &= K_o = 1 \end{aligned} \tag{7}$$

The parameters K_c, T_i and K_o have been adjusted by user, while K_f follows from equations:

$$\alpha = \sqrt{\frac{\beta_0 K_c}{T_i}} = 0.5222 \tag{8}$$

$$c_1 = \alpha T_i = 0.05745 \Rightarrow d_1 = 1.3405 \tag{9}$$

$$K_f = \frac{d_1\alpha - \alpha_0 - K_c\beta_0}{\beta_0} \tag{10}$$

The size of d_1 in (9) must be determined on the basis of c_1 according to graph from (Kaya and Atherton 1999).

Besides, a non-zero value of K_o ensures better disturbance rejection, but there is trade-off between this rejection and oscillation of the control and output signal (see figs. 4 and 5). The behaviour would be "smoother" for $K_o = 0$.

In CDM, as the second method, the settling time was preset to $t_s = 5s$ and disturbance rejection structure was selected. The resulting controllers are:

$$\begin{aligned} G_{c1}(s) &= 1 \\ G_{c2}(s) &= \frac{1}{l_1 s} = \frac{1}{2.1557s} \\ G_{c3}(s) &= k_1 s + 1 = 0.1658s + 1 \end{aligned} \tag{11}$$

The coefficients of regulators follow from:

$$l_1 = \frac{K\tau^2}{2.5T} \tag{12}$$

$$k_1 = \tau - \frac{\tau^2}{2.5T} \tag{13}$$

where

$$\tau = t_s/2.1538 \tag{14}$$

and transfer function of controlled system model (without TD) is assumed in the form:

$$G_m(s) = \frac{K}{Ts+1} = \frac{1}{s+1} \tag{15}$$

Regarding to the third technique, control design in R_{PS}, the nominal system is obtained using the first order Padé approximation of TD in (3):

$$\begin{aligned} G(s) &= \frac{1}{s+1}e^{-s} \approx \frac{1-0.5s}{(s+1)(1+0.5s)} = \\ &= \frac{-s+2}{s^2+3s+2} = \frac{b_1 s + b_0}{s^2 + a_1 s + a_0} \end{aligned} \tag{16}$$

The choice $m = 1.2$ gives the feedback (1DOF) controller:

$$\begin{aligned} C_b(s) &= \frac{\tilde{q}_2 s^2 + \tilde{q}_1 s + \tilde{q}_0}{s^2 + \tilde{p}_1 s} = \\ &= \frac{0.5691s^2 + 1.6053s + 1.0368}{s^2 + 2.3691s} \end{aligned} \tag{17}$$

Its parameters are calculated from equations:

$$\tilde{p}_1 = p_0 + m - p_0 m \frac{b_1}{b_0}$$
$$\tilde{q}_2 = q_1 + \frac{p_0 m}{b_0}$$
$$\tilde{q}_1 = q_0 + q_1 m + a_1 \frac{p_0 m}{b_0} \tag{18}$$
$$\tilde{q}_0 = q_0 m + a_0 \frac{p_0 m}{b_0}$$

and

$$p_1 = 1$$
$$p_0 = \frac{3m^2 b_0 b_1 - a_0 b_0 b_1 - 3m b_0^2 + a_1 b_0^2 - b_1^2 m^3}{a_1 b_0 b_1 - b_0^2 - a_0 b_1^2}$$
$$q_1 = \frac{3m - a_1 - p_0}{b_1} \tag{19}$$
$$q_0 = \frac{m^3 - a_0 p_0}{b_0}$$

The comparison of closed-loop output variables for all methods is shown in fig. 4 while related control signals are depicted in fig. 5.

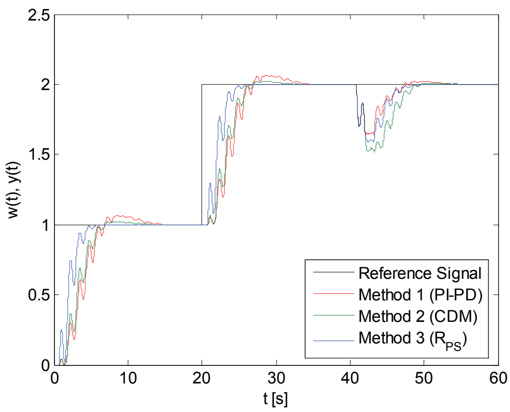

Fig. 4. Comparison of closed-loop control responses

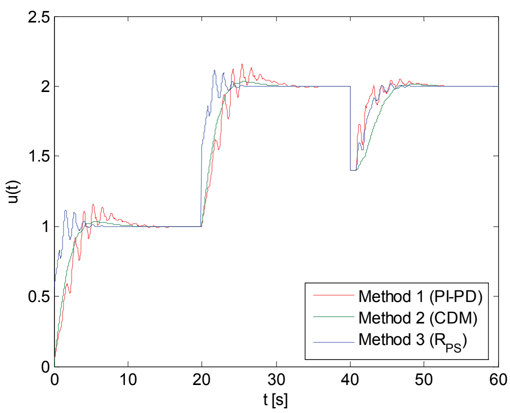

Fig. 5. Comparison of control signals

As can be seen, all methods are able to control given time-varying delay system relatively acceptable. Control design in R_{PS} gives the fastest response, on the top of that without overshoot, and good rejection of load disturbance. The cost for it is a bit more aggressive control signal. The modified PI-PD Smith predictor provides the best disturbance rejection thanks to the disturbance controller mentioned in the previous sections.

The evaluation of control behaviours from the viewpoint of the integral squared error (ISE) criterion can be found in table 1.

Method	ISE
PI-PD	6.115
CDM	5.989
R_{PS}	3.845

Table 1. Outcomes of ISE calculations

Besides, disadvantages of both modifications of Smith predictor are more complicated control loop structure and necessity of TD model in the inner loop. Hence, one can claim that the proposed control design in R_{PS} can be considered as an effective method for studied class of systems.

7 CONCLUSION

Three different continuous-time strategies based on the idea of robustness were compared during control of SISO systems with harmonically time-varying delay. The first two methods use the modified Smith predictor structures in combination with standard forms for minimum of ISTE or design by CDM, respectively. The third method is based on the fractional representation in R_{PS}, general solutions of Diophantine equations and conditions of divisibility. The simulations of control were done in Matlab + Simulink environment.

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