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DECENTRALIZED ROBUST CONTROL OF TWO INPUTS -TWO OUTPUTS SYSTEM

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Abstract: Decentralized control of interacting two input two outputs is presented in this paper. Robust control method is used for controllers design. Cross coupling circuits are taken as a model uncertainty. Approximation models are obtained experimentally by two relay autotuning experiments. Controllers are designed on the basis of common condition for robust stability and robust performance. The method is verified on the control simulation in the environment MATLAB/Simulink.

Key Words: Decentralized control, robust control, multivariable systems, uncertainty

1. INTRODUCTION

Control of multivariable systems with inner interactions can be realized either by the set of single-input single-output (SISO) decentralized controllers or by the centralized multiple-input multiple-output (MIMO) controller. This paper deals with methods of decentralized control, where the inner interactions are not suppressed as at decoupling, but they are taken into account by SISO controller tuning.

Various methods have been suggested for the tuning of decentralized controllers and this problem still pays attention in present control literature. Wide group of these methods are based on the determination of system stability limit, given by ultimate gain K_u and ultimate frequency ω_u (or period T_u) in every loop, as main information for SISO PID controllers tuning. The systems must be able to oscillate (must have phase lag greater then 180 degrees).

Courses of iterative experimental methods for decentralized control correspond with the mathematical solution of stability conditions. From this point of view three ways of solutions may be defined:

- Every loop has different ultimate gain and ultimate frequency (sequential method).
- All loops have the same ultimate frequencies (Palmor's method).
- All loops have the same ultimate gains and the same ultimate frequencies.

The detail information about principle these methods and their properties can be found *e.g.* in Macháček (2005).

Robust control methods are used for tuning of the decentralized controllers. Coupling part of model is taken as an additive uncertainty. Range of possible uncertainties is given by ultimate values determined according to Palmor's method. Controllers are designed on the basis of common condition for robust stability and robust performance. The method is first verified for known system model and then for approximation of model by relay feedback method.

2. ROBUST DECENTRALIZED CONTROL

Systems with two inputs and two outputs (TITO) will be only considered in this paper (Fig. 1). TITO system has the transfer function matrix

$$\boldsymbol{G}(s) = \begin{bmatrix} G_{11}(s) & \dots & G_{1n}(s) \\ \vdots & & \vdots \\ G_{n1}(s) & \dots & G_{nn}(s) \end{bmatrix} \quad (1)$$

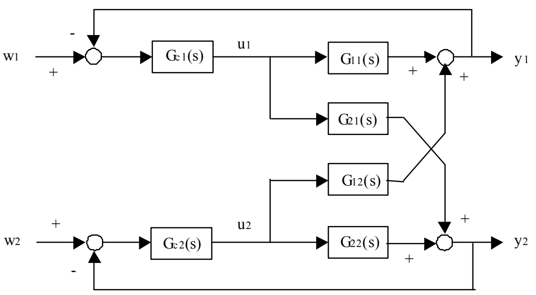

Fig. 1. Two-dimensional system with decentralized controllers

and is controlled by decentralized controller with diagonal matrix

$$\boldsymbol{G}_c(s) = \begin{bmatrix} G_{c1}(s) & 0 \\ 0 & G_{c2}(s) \end{bmatrix} \tag{2}$$

Pairing the controlled and manipulated variables is assumed to be correct. When the multivariable system is divided into SISO systems, transfer function of every loop depends not only on the partial diagonal transfer function, but also on the cross transfer functions and on the setting of controllers in rest loops. The controllers design must take this fact into account.

In case of TITO system the actual transfer function between the first input and the first output (without controller G_{c1} and with controller G_{c2} connected in the second loop) has the following form:

$$\overline{G}_{11}(s) = G_{11}(s) - \frac{G_{12}(s)G_{21}(s)G_{c2}(s)}{1+G_{22}(s)G_{c2}(s)} \tag{3}$$

The second transfer function with a controller in the first loop has similar form:

$$\overline{G}_{22}(s) = G_{22}(s) - \frac{G_{12}(s)G_{21}(s)G_{c1}(s)}{1+G_{11}(s)G_{c1}(s)} \tag{4}$$

The described method considers diagonal transfer functions $G_{11}(s)$ and $G_{22}(s)$ as a nominal model and the second coupling parts in Eqs. (3) and (4) as a model uncertainty. The set of possible controller parameters creates family of models, for which a robust controller is designed. Range of the possible controllers parameters in Eqs. (3) and (4) was derived on the basis of above mentioned Palmor's method of decentralized control – Palmor et al. (1995). At this method relays are connected in both loops simultaneously and loops have the same ultimate frequencies. There are infinite numbers of the combinations of ultimate values, according to relay amplitude ratio. Some additional condition must be chosen, *e.g.* the same value of product of ultimate gain and steady-state gain for both loops. The relay amplitude ratio can be found, for which the condition is fulfilled after several iterations. Stability limit for two-dimensional system is a curve in the K_{u1},K_{u2} plane, where points on the axes represent ultimate gains for system without controllers, *i.e.* for diagonal transfer functions $G_{11}(s)$ and $G_{22}(s)$. The ultimate gains and frequencies both transfer functions are also limit values for controller's design. Roughly speaking the ultimate gain of the diagonal transfer functions may be taken as maximum values for all possible controllers and difference between ultimate frequencies gives the range all possible ultimate frequencies used for controllers design – see Fig. 2.

PID controller is used in both loops. Its transfer function has following basic form

$$G_c(s) = K\left(1 + \frac{1}{T_i s} + T_d s\right) \tag{5}$$

where K is proportional gain, T_i is integral time and T_d is derivative time. The PID controller parameters design is realized by Ziegler-Nichols (Z-N) frequency response method (Ziegler et al. (1942)) from the ultimate values. The classical Z-N formula gives these recommendations for PID controllers tuning

$$K = 0.6\,K_u, \quad T_i = 0.5\,T_u, \quad T_d = 0.125\,T_u \tag{6}$$

2.1 *Uncertainty of model*

The uncertainty of model was described by additive perturbation model as it correspondes with uncertainties in Eqs. (3) and (4):

$$\vec{G}(s) = G(s) + W(s)\Delta(s) \tag{7}$$

where $\vec{G}(s)$ is a perturbed plant transfer function, $G(s)$ is a nominal plant transfer function, $W(s)$ is a weighting transfer function and $\Delta(s)$ is a variable stable transfer function satisfying

$$\|\Delta\|_\infty < 1 \tag{8}$$

Comparison of Eqs. (3) and (4) with Eq. (7) gives

$$W_1(s)\Delta_1(s) = -\frac{G_{12}(s)G_{21}(s)G_{c2}(s)}{1+G_{22}(s)G_{c2}(s)} \tag{9}$$

$$W_2(s)\Delta_2(s) = -\frac{G_{12}(s)G_{21}(s)G_{c1}(s)}{1+G_{11}(s)G_{c1}(s)} \tag{10}$$

The perturbance weighting functions may be calculated on condition (8) as

$$W_1(i\omega) \geq -\frac{G_{12}(i\omega)G_{21}(i\omega)G_{c2}(i\omega)}{1+G_{22}(i\omega)G_{c2}(i\omega)} \qquad \forall\omega \qquad (11)$$

$$W_2(i\omega) \geq -\frac{G_{12}(i\omega)G_{21}(i\omega)G_{c1}(i\omega)}{1+G_{11}(i\omega)G_{c1}(i\omega)} \qquad \forall\omega \qquad (12)$$

For the next calculation must be these weighting functions approximated by simpler transfer functions. The perturbances, defined in this way for all possible contollers in the opposite loop, faciliate the independent design both controllers.

2.2 *Robust stability*

The robust stability condition for additive perturbance model is given (*e.g.* Skogestad (1995))

$$\left|S_m(i\omega)W_m(i\omega)G_{cm}(i\omega)\right| < 1 \quad \forall\omega,\ m = 1, 2 \quad (13)$$

where S(s) is sensitivity function

$$S_m(s) = \frac{1}{1+G_{mm}(s)G_{cm}(s)} \qquad m = 1, 2 \qquad (14)$$

This condition reduces range of possible controllers.

2.3 *Robust performance*

The sensitivity function was taken as indicator of closed-loop performance. Reciprocal values of chosen weighting function W_p(s) are an upper bound of the amplitude of sensitivity function

.

$$\left|S(i\omega)\right| < \frac{1}{\left|W_p(i\omega)\right|} \qquad \forall\omega \qquad (15)$$

where $W_p(s)$ is weight, which may be represented by

$$W_p(s) = \frac{s/M+\omega_B}{s+\omega_B A} \qquad (16)$$

where A is low frequencies amplitude of $S(i\omega)$, M is high frequencies amplitude of $S(i\omega)$ and ω_B is frequency, where amplitude crosses 1 from below.

The common condition for robust stability and robust performance may be expressed as

$$\left|S_m(i\omega)W_{pm}(i\omega)\right| + \left|S_m(i\omega)W_m(i\omega)G_{cm}(i\omega)\right| < 1 \quad \forall\omega$$

$$m = 1, 2 \qquad (17)$$

2.4 *Approximation of model*

The exact system model is not usually known and simple approximating model may be created by relay experiments. The experiment consists of two parts: The relay is firstly connected between the first input and the first output and the responses of transfer functions $G_{11}(s)$ and $G_{21}(s)$ are measured and evaluated. Then relay is transferred between the second input and the second output and the responses of transfer functions $G_{22}(s)$ and $G_{12}(s)$ are investigated. Two models may be derived from every experiment. Gains of models are determined from steady states.

The relay with amplitude $\pm M$ in the feedback generates output signal with amplitude A and period T_u, which is near to the sinusoidal. The ultimate gain can be then calculated as

$$K_u = \frac{4M}{\pi A} \qquad (18)$$

and ultimate period T_u is determined from recorded signals. Ultimate frequency is given by

$$\omega_u = \frac{2\pi}{T_u} \qquad (19)$$

Ultimate gains (K_{u11} and K_{u22}) and ultimate frequencies (ω_{u11} and ω_{u22}) of diagonal transfer functions are the basis for determination of approximation models G_{a11} and G_{a22} (*e.g* Macháček (1998)):

$$\left|G_{a11}(i\omega_{u11})\right| = \frac{1}{K_{u11}} \qquad (20)$$

$$\left|G_{a22}(i\omega_{u22})\right| = \frac{1}{K_{u22}} \qquad (21)$$

$$\arg\left[G_{a11}(i\omega_{u11})\right] = \arg\left[G_{a22}(i\omega_{u22})\right] = -\pi \qquad (22)$$

The models of cross transfer functions are solved similarly, but gains and frequencies are not ultimate and shift phases (different from $-\pi$) are determined from experiment.

$$\left|G_{a21}(i\omega_{u11})\right| = \frac{1}{K_{u21}} \qquad (23)$$

$$\left|G_{a12}(i\omega_{u22})\right| = \frac{1}{K_{u12}} \qquad (24)$$

$$\arg\left[G_{a21}(i\omega_{u11})\right] = -\varphi_{21} \qquad (25)$$

$$\arg\left[G_{a12}(i\omega_{u22})\right] = -\varphi_{12} \qquad (26)$$

These approximation models are not always accurate, but their uncertainty is added to model uncertainty.

2.5 *Design algorithm*

1. Experiment with relay feedback is repead for both loops. Approximate models are evaluated from measured data. The ultimate values at the same time determine set of possible controllers.

2. For given range of possible controllers are calculated addditive weights and find their approximation.

3. Shapes of sensitivity functions are designed.

4. Controllers designed from ultimate values according to Eq. (6) must fulfil the condition of robust stability and robust performance (17).

3. SIMULATION EXAMPLE

The method was tested in MATLAB/Simulink on model used in Niederlinski (1971):

$$G(s)=\frac{1}{(0.1s+1)(0.2s+1)^2}\begin{bmatrix}\frac{0.5}{(0.1s+1)} & -1\\ 1 & \frac{2.4}{(0.5s+1)}\end{bmatrix} \quad (27)$$

The method was first verified for accurate model. The course of ultimate gains and the common ultimate frequency calculated from model (27) are given in Fig. 2. The range of the ultimate values for controller parameters design is also:

$$K_{u11}=0 \text{ to } 9.00 \qquad K_{u22}=0 \text{ to } 2.28$$

$$\omega_u=4.33 \text{ to } 6.83\ s^{-1}$$

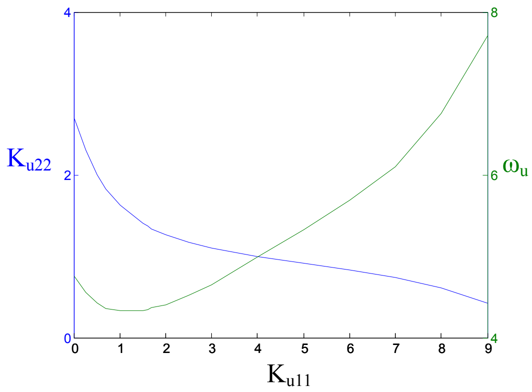

Fig. 2. Ultimate values of model (27)

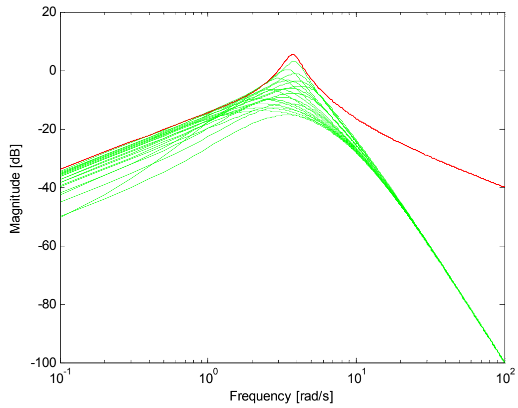

Fig. 3. Additive perturbations $W_1(i\omega)$ and their approximation by weighting function $W_{1a}(i\omega)$

Weighting functions W_1 and W_2 were calculated from Eqs. (11) and (12) for given range of opposite controllers. The nominal models had modified gains (multiplied by constant 1.833) in order that the weighs had smaller values. The weighting functions were approximated by following transfer functions:

$$W_{1a}(s)=\frac{0.2s\,(s^2+12s+14)}{(s^2+s+14)(0.2s+1)(s+1)}$$

$$W_{2a}(s)=\frac{(s+0.0001)}{(0.5s+0.1)(0.1s+1)(0.1s+1)(0.1s+1)}$$

Course of weighting functions and their approximations are shown on Figs. 3 and 4.

The weighing functions for closed loop performance according to Eq. (15) were designed as transfer function (16) with following parameters:

$$W_{p1}(s)=\frac{0.5s+0.15}{s+0.0015}$$

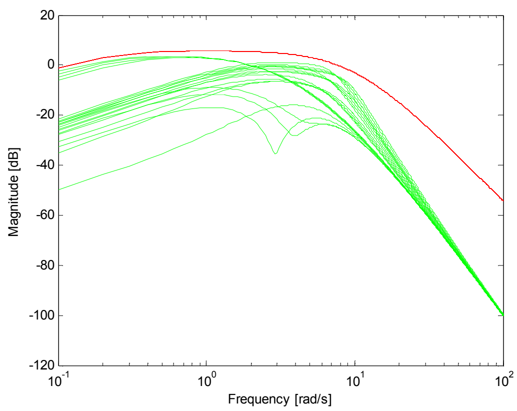

Fig. 4. Additive perturbations $W_2(i\omega)$ and their approximation by weighting function $W_{2a}(i\omega)$

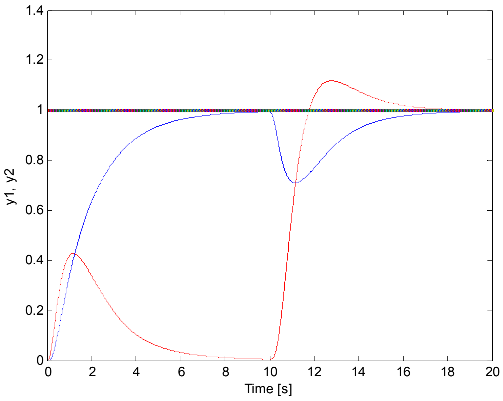

Fig. 5 – Closed-loop step responses (y_1 - blue, y_2 - red)

$$W_{p2}(s) = \frac{0.1s + 0.2}{s + 0.002}$$

Condition on robust stability and robust performance (17) are fulfilled for following controller setting:

$K_1 = 0.35$ $K_2 = 0.24$
$T_{i1} = T_{i2} = 0.65$ s
$T_{d1} = T_{d2} = 0.16$ s

Closed-loop responses to unit step in the first reference in time 0 s and in the second one in time 10 s are shown on Fig. 5.

Original Palmor's method gives responses according to Fig. 6.

3.1 *Model approximation*

Experiment with relay feedback gives less accurate values:

$K_{u11} = 8.33$ $K_{u22} = 2.13$
$\omega_{u11} = 6.83$ s^{-1} $\omega_{u22} = 4.62$ s^{-1}
$K_{u12} = -1.95$ $K_{u21} = 3.39$
$\omega_{u12} = 4.62$ s^{-1} $\omega_{u21} = 6.83$ s^{-1}

Approximate models were chosen in the general form

$$G_a(s) = \frac{K}{(\tau s + 1)^2} e^{-\theta s} \qquad (28)$$

with the following parameters:

Gains
$K_{11} = 0.5$ $K_{12} = -1$
$K_{21} = 1$ $K_{22} = 2.4$
Time constants
$\tau_{11} = 0.26$ s $\tau_{12} = 0.23$ s
$\tau_{21} = 0.22$ s $\tau_{22} = 0.47$ s

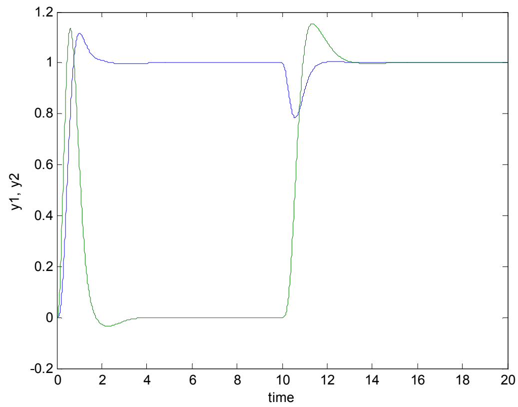

Fig. 6 – Closed-loop step responses – Palmor's method (y_1 - blue, y_2 - green)

Time delays
$\theta_{11} = 0.10$ s $\theta_{11} = 0.05$ s
$\theta_{21} = 0.05$ s $\theta_{22} = 0.10$ s

Model parameters were computed by optimization procedure.

The parameters of controllers as well as the time responses of approximate models did not too differ from accurate model.

4. CONCLUSION

Method of decentralized control, based on robust control method, was described and verified in this paper. Advantage this method is in shorter experimental work in comparison with the classic Palmor's method, as only two relay experiment is claimed instead of several iterations. The results of simulation in environment MATLAB/Simulink show, that responses are slower, but the responses overshoots are smaller.

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